

Books, notes (in any form), calculators, etc., are not allowed. You must *show all your work* for full credit. Good Luck!

1. (15 pts) Given the following function and its derivatives

$$f(x) = \frac{2x^2}{x^2 + 3}$$

$$f'(x) = \frac{12x}{(x^2 + 3)^2}$$

$$f''(x) = \frac{-36(x^2 - 1)}{(x^2 + 3)^3}$$

(a). Fill in the following information about the function and its graph. Show all work and write "none", if applicable.

$$x^2 + 3 = 0 \text{ Never} \Rightarrow \text{Domain: All real \#s}$$

domain:

All real numbers

x-intercept(s):

pt. (0, 0)

$$x\text{-int (y=0): } 2x^2 = 0 \Rightarrow x = 0$$

$$y\text{-int (x=0): } f(0) = \frac{0}{0+3} = 0$$

$$V.A. \ x^2 + 3 = 0 \text{ Never}$$

y-intercept:

pt. (0, 0)

$$H.A. \ \lim_{x \rightarrow \infty} \frac{2x^2}{x^2 + 3} = 2 \quad \lim_{x \rightarrow -\infty} \frac{2x^2}{x^2 + 3} = 2$$

vertical

asymptote(s):

None

$$f'(x) = 0$$

$$f'(x) \text{ DNE}$$

$$12x = 0$$

$$(x^2 + 3)^2 = 0$$

$$x = 0$$

not possible

horizontal

asymptote(s):

y = 2

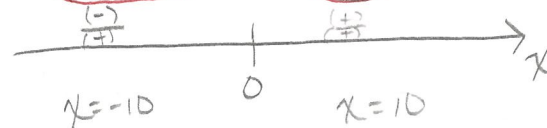
slant asymptote:

None

critical numbers:

x = 0

f'



intervals where increasing:

(0, ∞)

Local min at x = 0 : f(0) = 0

intervals where decreasing:

(-∞, 0)

coordinates of local max(s):

None

coordinates of local min(s):

(0, 0)

intervals where concave up:

(-1, 1)

intervals where concave down:

(-∞, -1) ∪ (1, ∞)

f''



Inflection Point(s):

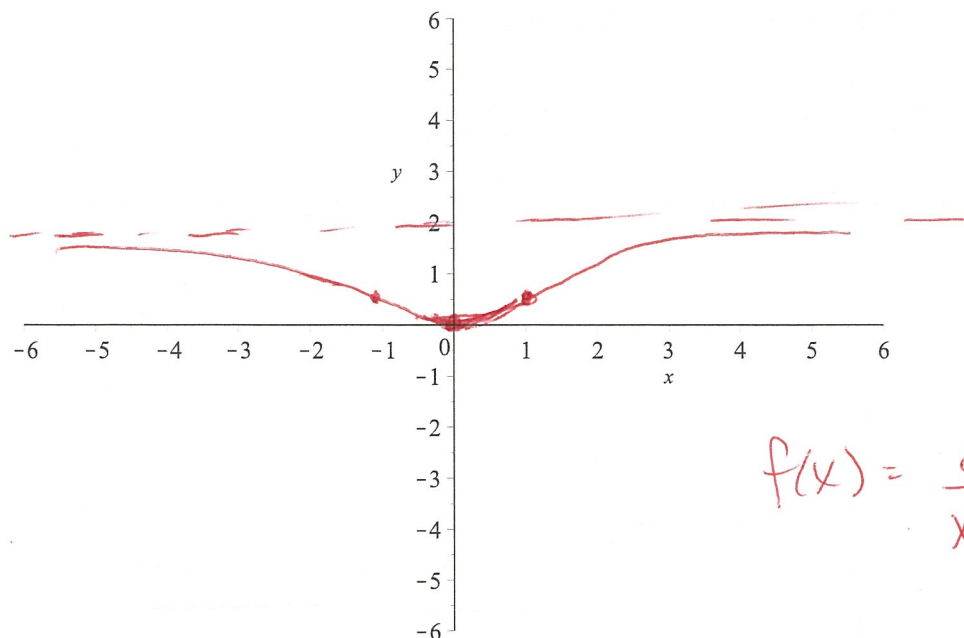
(1, 1/2) & (-1, 1/2)

Do I's at x = ±1

$$f(1) = \frac{2}{1+3} = \frac{2}{4} = \frac{1}{2}$$

$$f(-1) = \frac{2(-1)^2}{(-1)^2 + 3} = \frac{2}{4} = \frac{1}{2}$$

- (b). Sketch the graph of the function on the set of axes provided. Label any maximum and minimum values and inflection points.



Just an extra set of axes, in case you need it.

