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1. Find the antiderivatives for the following functions.

(a). $h(x) = 3x^3 - 7x^2$ $H(x) = \frac{3}{4}x^4 - \frac{7}{3}x^3 + C$

(b). $f(x) = \sqrt{x} - \sqrt{3}x^2$ $F(x) = \frac{2}{3}x^{3/2} - \frac{\sqrt{3}}{3}x^3 + C$

2. Given that $g'(\theta) = -\sec^2\theta$ and $g\left(\frac{\pi}{3}\right) = 0$, find $g(\theta)$. $g(\theta) = -\tan\theta + \sqrt{3}$

3. Given the function $f(x) = \frac{3}{x}$, estimate the area under the curve $f(x)$ on the interval $[1, 6]$ using 5 subintervals and using the right endpoint of each subinterval. [i.e. find R_5, but do NOT simplify.].
$R_5 = \Delta x \cdot [f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] = 1 \cdot [f(2) + f(3) + f(4) + f(5) + f(6)] = \boxed{1 \cdot [3/2 + 3/3 + 3/4 + 3/5 + 3/6]}$

4. Using the definition of the definite integral $\int_a^b f(x)\ dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} R_n$, ***set-up, but do not evaluate***, the summation/limit using right endpoints for the integral $\int_0^1 x^3 + 1\ dx$.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left[\left(\frac{i}{n}\right)^3 + 1\right]$$

5. Evaluate the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(\frac{i}{n}\right)^2 = \frac{1}{3}$

6. Section 4.3: #3

7. Evaluate the following integrals [Use integration techniques, ***not*** the limit definition.]:

(a). $\int_1^2 t + 2\ dt = \frac{7}{2}$

(b). $\int_1^{x^2} t + 2\ dt = \frac{1}{2}x^4 + 2x^2 - \frac{5}{2}$

(c). $\int_0^4 \frac{x(2+x)}{\sqrt{x}}\ dx = \frac{352}{15}$

8. Use the Fundamental Theorem of Calculus (Part B/1) to find $F'(x)$

(a). $F(x) = \int_0^x t\cos t\ dt$ $F'(x) = x\cos x$

(b). $F(x) = \int_{-2}^{x^2}\sqrt{t+8}\ dt$ $F'(x) = \sqrt{x^2+8}\cdot 2x = 2x\sqrt{x^2+8}$

9. A particle moves with a velocity of $v(t) = -t^2 + 4t$ on the interval $0 \le t \le 6$.

(a). Find the displacement 0

(b). Find the total distance traveled $\frac{32}{3} + \left|-\frac{32}{3}\right| = \frac{64}{3}$

10. Let $r(t)$ be the rate at which the world's oil is consumed, where t is measured in years starting at $t = 0$ on January 1, 2000 and $r(t)$ is measured in barrels per year. What does $\int_0^8 r(t)\ dt$ represent and what are its units?
The integral represents the change in the amount of oil consumed from 2000 to 2008. The units are are barrels.

11. Evaluate the following integrals. [**Note:** You may or may not need to use substitution.] Check your answer by differentiating the result.

(a). $\int_0^2 t^2\sqrt{1+t^3}\,dt = \frac{52}{9}$ u-substitution

(b). $\int \sin x\cos(\cos x)\,dx = -\sin(\cos x) + C$ u-substitution

(c). $\int 3x^5 - 4x^3 + 6x + 2\,dx = \frac{1}{2}x^6 - x^4 + 3x^2 + 2x + C$ direct integration

(d). $\int (3x-1)(3x^2-2x)^2\,dx = \frac{1}{6}(3x^2-2x)^3 + C$ u-substitution

(e). $\int x(3x^2-2x)^2\,dx = \frac{3}{2}x^6 - \frac{12}{5}x^5 + x^4 + C$ expand/simplify

(f). $\int \left(1+\frac{1}{t}\right)\left(\frac{1}{t^2}\right)\,dt = -\frac{1}{t} - \frac{1}{2t^2} + C$ multiply/simplify OR u-substitution

(g). $\int_0^{\pi/6} \sec x\tan x\,dx = \frac{2}{\sqrt{3}} - 1$ direct integration rule

(h). $\int \sin x\cos x\,dx = \frac{1}{2}\sin^2 x + C$ OR $-\frac{1}{2}\cos^2 x + C$ u-substitution

(i). $\int \frac{5x}{\sqrt[3]{1-x^2}}\,dx = -\frac{15}{4}(1-x^2)^{2/3} + C$ u-substitution

(j). $\int_1^3 \frac{x^2+1}{x^2}\,dx = \frac{8}{3}$ simplify

(k). $\int y^2\sqrt{y}\,dy = \frac{2}{7}y^{7/2} + C$ simplify

(l). $\int_0^1 (2-x)^6\,dx = \frac{127}{7}$ u-substitution

(m). $\int \theta\sin(3\theta^2)\,d\theta = -\frac{1}{6}\cos(3\theta^2) + C$ u-substitution

12. Sketch the region bounded by the graphs of the following functions. Find the ***area*** of the region.

(a). $f(x) = 3 - 2x - x^2, \;\; g(x) = -x + 1$ $\frac{9}{2}$

(b). $x = y^2, \;\; x = -y$ $\frac{1}{6}$

13. Set up, but do *not* evaluate the integral(s) to find ***volume*** of the solid generated by rotating the region bounded by the given curves about the given line.

(a). $y = x^2, y = 4x - x^2$ about the line $y = 6$

$$V = \int_0^2 \pi \left[(6 - x^2)^2 - (6 - 4x + x^2)^2\right] \, dx$$

(b). $xy = 6, y = 2, y = 6, x = 6$ about the line $x = 6$.

$$V = \int_2^6 \pi \left(6 - \frac{6}{y}\right)^2 \, dy$$

14. The force exerted by gravity on an object sent into space is given by $F(x) = \dfrac{4.8 \times 10^{11}}{x^2}$ pounds where x is measured in miles from the *center* of the earth. How much work is done to propel a satellite module to 800 miles above the earth. Use 4000 miles for the radius of the earth. [Similar problems not requiring a calculator may be on the test.]

$$W = \int_{4000}^{4800} 4.8 \times 10^{11} x^{-2} \, dx = 2 \times 10^7 \text{ mile}\cdot\text{ lbs} = 1.056 \times 10^{11} \text{ foot}\cdot\text{ lbs}$$

15. If 18 J of work is required to stretch a spring 40 cm from it's natural length, find the work required to stretch it an additional 30 cm.

Convert units to m. $k = 225 \Rightarrow W = 37.125$ J

16. Given $f(x) = \dfrac{4x^2 + 4}{x^2}$

(a). Find the average value of $f(x)$ on the interval $[-3, -1]$. $f_{ave} = \frac{16}{3}$

(b). Use the Mean Value Theorem for integrals to find all values $x = c$ where $f(c) = f_{ave}$. $x = -\sqrt{3}$

17. Find the value of k so that the average value of $f(x) = kx^2 - x$ on [0,2] is equal to 4. $k = \frac{15}{4}$

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