[Note: Section 2.8 Related Rates will ***not*** be on this Exam.]

1. Find the critical numbers for $g(t) = 4t^3 - 3t^2 + 1$ $t = 0, \frac{1}{2}$

2. Given $f(x) = 2\sqrt{x} - x$, find the absolute maximum and minimum *values* of $f(x)$ on the interval $[0, 9]$.
Absolute Max $= 1$ at $x = 1$; Absolute Min $= -3$ at $x = 9$.

3. Let $f(x) = 1 - x^{2/3}$. Show that $f(-1) = f(1)$ but there is no number c in $(-1, 1)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem? $f(-1) = 0$ and $f(1) = 0$ and $f'(x) = \frac{-2}{3x^{1/3}}$ which can never equal zero. The derivative does not exist at $x = 0$ (i.e., f is not differentiable on the entire interval $(-1, 1)$), so the conditions of Rolle's Theorem do not hold.

4. Apply the Mean Value Theorem to the function $f(x) = \sqrt{x-2}$ on the interval [2, 6] and find all values of c that satisfy the MVT. $c = 3$

5. Given $f(x) = \dfrac{(x-1)^3}{x^2}$

(a). Find the intervals of increase or decrease. Increasing: $(-\infty, -2) \cup (0, 1) \cup (1, \infty)$ Decreasing: $(-2, 0)$

(b). Find the local maximum and minimum values. No Min ; Max $= -\frac{27}{4}$ at $x = -2$.

(c). Find the intervals of concavity and the inflection points. Up: $(1, \infty)$ Down: $(-\infty, 0) \cup (0, 1)$ Inf. Pts.: (1,0)

6. Given $f(\theta) = \cos^2(\theta)$, on $0 \le \theta \le 2\pi$,

(a). Find the intervals of increase or decrease. Increasing: $\left(\frac{\pi}{2}, \pi\right) \bigcup \left(\frac{3\pi}{2}, 2\pi\right)$ Decreasing: $\left(0, \frac{\pi}{2}\right) \bigcup \left(\pi, \frac{3\pi}{2}\right)$

(b). Find the local maximum and minimum values. Min $= 0$ at $x = \frac{\pi}{2}, \frac{3\pi}{2}$ and Max $= 1$ at $x = \pi$.

(c). Find the intervals of concavity and the inflection points.
Up: $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \bigcup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$ Down: $\left(0, \frac{\pi}{4}\right) \bigcup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \bigcup \left(\frac{7\pi}{4}, 2\pi\right)$ Inf. Pts.: $\left(\frac{\pi}{4}, \frac{1}{2}\right), \left(\frac{3\pi}{4}, \frac{1}{2}\right), \left(\frac{5\pi}{4}, \frac{1}{2}\right), \left(\frac{7\pi}{4}, \frac{1}{2}\right)$

7. Section 3.3 #27

8. Evaluate the following limits. [Show all work - no shortcuts].

(a). $\displaystyle\lim_{x\to\infty} \frac{3 - x^2 + 4x^3}{x^4 + 2x} = 0$

(b). $\displaystyle\lim_{x\to-\infty} \frac{2x^2 + 3x + 1}{3x^2 - 3x - 4} = \frac{2}{3}$

9. Find $\displaystyle\lim_{x\to\pm\infty} f(x)$ for the following functions and determine any horizontal and slant asymptotes.

(a). $f(x) = \dfrac{2x+1}{\sqrt{4x^2 - x}}$

(b). $g(x) = \dfrac{2x^3 - 3x^2 + 2}{x^2 - 3x}$

(a). $\displaystyle\lim_{x\to\infty} f(x) = 1$; $\displaystyle\lim_{x\to-\infty} f(x) = -1$ HA: $y = 1$ and $y = -1$

(b) $\displaystyle\lim_{x\to\infty} g(x) = \infty$; $\displaystyle\lim_{x\to-\infty} g(x) = -\infty$ SA: $y = 2x + 3$

10. Given the following function and its derivatives

$$f(x) = \frac{x}{9-x^2} \qquad f'(x) = \frac{x^2+9}{(9-x^2)^2} \qquad f''(x) = \frac{2x(x^2+27)}{(9-x^2)^3}$$

Use the Summary of Curve Sketching to determine the relevant information. Sketch the graph of the function. Label any maximum and minimum points and inflection points.

domain: All real numbers, except $x \neq \pm 3$

x-intercept(s): $x = 0$

y-intercept: $y = 0$

vertical asymptote(s): $x = 3, x = -3$

horizontal asymptote(s): $y = 0$

slant asymptote: none

critical numbers: none

intervals where increasing: $(\infty, -3) \bigcup (-3, 3) \bigcup (3, \infty)$

intervals where decreasing: none

coordinates of local max/min(s): none

intervals where concave up: $(\infty, -3) \bigcup (0, 3)$

intervals where concave down: $(-3, 0) \bigcup (3, \infty)$

inflection point(s): $(0, 0)$

Use your calculator to graph the function to check and see if you sketched it correctly.

11. If a box with a square base and open top is to hold 4 ft^3, find the dimensions of the box that will require the least amount of material.

$2' \times 2' \times 1'$

12. Find the maximum possible volume of a right circular cylinder if its total surface area (including top and bottom) is 150π.

$V = 250\pi \qquad (r = 5, h = 10)$

13. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions of the Norman window with the largest possible area if the total perimeter is 16 ft. Rect. portion: $\frac{16}{\pi+4} \times \frac{32}{\pi+4}$ ft.

14. Given $f(x) = 4x^3 - 12x^2 + 12x - 3$, [The graph is given below.]

(a). Explicitly write out Newton's formula for finding the root of this function.

$$x_{n+1} = x_n - \frac{4x_n^3 - 12x_n^2 + 12x_n - 3}{12x_n^2 - 24x_n + 12}$$

(b). Start with an initial guess of $x_0 = 0.5$ and iterate Newton's Method once. [***Do <u>not</u> simplify your answer!***]

$$x_1 = 0.5 - \frac{4(0.5)^3 - 12(0.5)^2 + 12(0.5) - 3}{12(0.5)^2 - 24(0.5) + 12}$$

(c). Starting with $x_0 = 0.5$, demonstrate Newton's method by marking $x_0, x_1, x_2, \ldots$ and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess? Yes

(d). Starting with $x_0 = 1.0$, demonstrate Newton's method by marking $x_0, x_1, x_2, \ldots$ and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess? Why or why not?

No, it won't work since the tangent line is horizontal when $x_0 = 1.0$.