Name: Key

Exam 3-A/B/C

Math 151-01, Calculus I – Crawford

14 November 2017

Score	
1	/12
2	/14
3	/12
4	/14
5	/8
6	/12
7	/8
8	/8
9	/14
Total	/100

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- You may use the provided unit circle.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- ***Good luck***!

1. (12 pts). Apply the Mean Value Theorem to the function $f(x) = \frac{1}{x}$ on the interval $[1, 3]$ and find all values of c that satisfy the MVT.

f is cont. on $[1,3]$ ✓

f is diff. on $(1,3)$ ✓

$$\frac{f(3)-f(1)}{3-1} = \frac{\frac{1}{3}-\frac{1}{1}}{2} = \frac{\frac{1}{3}-\frac{3}{3}}{2} = \frac{\left(\frac{-2}{3}\right)}{2}$$

$$= -\frac{2}{3}\cdot\frac{1}{2} = -\frac{1}{3}$$

$$f'(x) = -\frac{1}{x^2}$$

Set equal & solve for x

$$-\frac{1}{x^2} = -\frac{1}{3}$$

$$x^2 = 3$$

$x = \pm\sqrt{3}$ but only $\boxed{x=\sqrt{3}}$ is in the interval $[1,3]$.

2. (14 pts). Find the **absolute** maximum and minimum **values** of $f(x) = (x^2 - 1)^3$ on the interval $[-1, 2]$.

$f'(x) = 3(x^2-1)^2 \cdot 2x = 0$ (f' exists everywhere)

$6x(x^2-1)^2 = 0$

crit #'s $\longrightarrow$

$6x = 0$ or $x^2 - 1 = 0$

$x = 0$ $\quad (x-1)(x+1) = 0$

$x = 1, -1$

Evaluate f at the critical numbers & endpts.

crit #'s

$f(0) = (0^2-1)^3 = (-1)^3 = \boxed{-1 \leftarrow \text{Absolute Min}}$

$f(1) = (1^2-1)^3 = 0$

endpts

$f(-1) = ((-1)^2-1)^3 = (1-1)^3 = 0$

$f(2) = (2^2-1)^3 = (4-1)^3 = 3^3 = \boxed{27 \leftarrow \text{Absolute Max}}$

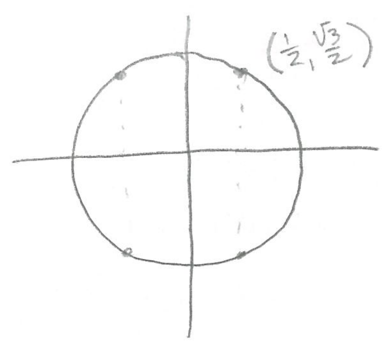
3. (12 pts). Find the critical numbers of $g(\theta) = 4\theta - \tan\theta$ for $0 \le \theta \le 2\pi$.

$g'(\theta) = 4 - \sec^2\theta = 0$

$\sec^2\theta = 4$

$\sec\theta = \pm 2$

$\Rightarrow \cos\theta = \pm\frac{1}{2}$



$(\frac{1}{2}, \frac{\sqrt{3}}{2})$ $\theta = \frac{\pi}{3}$

$$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

4. (14 pts). Given $f(x) = \dfrac{4x-5}{x^2}$,

(a). Find the intervals on which f is increasing or decreasing.

$$f'(x) = \frac{x^2 \cdot 4 - (4x-5)\cdot 2x}{(x^2)^2} = \frac{4x^2 - 8x^2 + 10x}{x^4}$$

$$= \frac{-4x^2+10x}{x^4} = \frac{-2\cancel{x}(2x-5)}{x^{\cancel{4}3}} = \frac{-2(2x-5)}{x^3} = f'(x)$$

Test

$\underline{f'(x) = 0}$

$-2(2x-5) = 0$

$2x = 5$

$x = \frac{5}{2}$

$\underline{f'(x) \text{ DNE}}$

$x^3 = 0$

$x = 0$

(Asymptote)

$x=-10$	$x=1$	$x=10$
$\frac{(-)(-)}{(-)}$	$\frac{(-)(-)}{(+)}$	$\frac{(-)(+)}{(+)}$
$-$	$+$	$-$

0 (asymptote), $\frac{5}{2}$

Asymptote

Increasing on $(0, \frac{5}{2})$

Decreasing on $(-\infty, 0) \cup (\frac{5}{2}, \infty)$

(b). Find all local maximum and minimum ***values***.

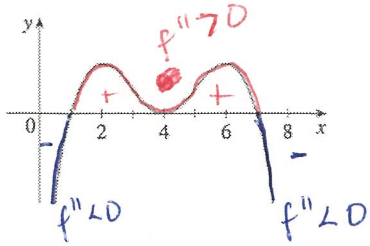
Local max at $x = \frac{5}{2}$:

$$f\left(\tfrac{5}{2}\right) = \frac{\cancel{4}^{2} \cdot \frac{5}{\cancel{2}} - 5}{\left(\frac{5}{2}\right)^2} = \frac{10-5}{\left(\frac{25}{4}\right)} = \frac{5}{\left(\frac{25}{4}\right)} = \frac{\cancel{5}}{1} \cdot \frac{4}{\cancel{25}_{5}}$$

$= \boxed{\frac{4}{5}}$ ← Local max value.

No local mins.

5. (8 pts). The graph of the ***second derivative*** $f''(x)$ is given below.



ie f'': $-$ | 1 | $+$ | 4 | $+$ | 7 | $-$ → X

(a). On which intervals is the ***function*** f (not the second derivative) concave upward or concave downward?

Upward: $(1,4)\cup(4,7)$
Downward: $(-\infty,1)\cup(7,\infty)$

(b). State the x-coordinate(s) of the inflection point(s) of the ***function*** f.

Changes concavity at $\boxed{x = 1, 7}$

6. (12 pts). Given the equation $\cos x = 3x^2 - x$.

(a). Explicitly write out Newton's formula for finding the root of this equation.

$\underbrace{\cos x - 3x^2 + x}$ $= 0$

$f(x) = \cos x - 3x^2 + x$

$f'(x) = -\sin x - 6x + 1$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\boxed{x_{n+1} = x_n - \frac{\cos x_n - 3x_n^2 + x_n}{-\sin x_n - 6x_n + 1}}$$

(b). Start with an initial guess of $x_0 = 1.5$ and iterate Newton's method to find x_1. *[Do not simplify your answer!]*

$$\boxed{x_1 = 1.5 - \frac{\cos(1.5) - 3(1.5)^2 + 1.5}{-\sin(1.5) - 6(1.5) + 1}}$$

7. (8 pts). Evaluate the following limit. [Show all your work - no shortcuts.]

$$\lim_{x \to -\infty} \frac{4 - 3x - 2x^3}{3x^3 - x + 1} = \lim_{x \to -\infty} \frac{\cancel{x^3}\left(\frac{4}{x^3} - \frac{3}{x^2} - 2\right)}{\cancel{x^3}\left(3 - \frac{1}{x^2} + \frac{1}{x^3}\right)}$$

$$= \lim_{x \to -\infty} \frac{\cancel{\frac{4}{x^3}}_{\to 0} - \cancel{\frac{3}{x^2}}_{\to 0} - 2}{3 - \cancel{\frac{1}{x^2}}_{\to 0} + \cancel{\frac{1}{x^3}}_{\to 0}}$$

$$= \boxed{-\frac{2}{3}}$$

8. (8 pts). Determine the slant asymptote of $f(x) = \frac{3x^2 + 1}{x - 2}$. [Do not sketch the curve.]

$$\begin{array}{r} 3x + 6 \\ x - 2 \,\big)\, 3x^2 + 0x + 1 \\ -3x^2 \pm 6x \\ \hline 6x + 1 \\ -6x \mp 12 \\ \hline 13 \end{array}$$

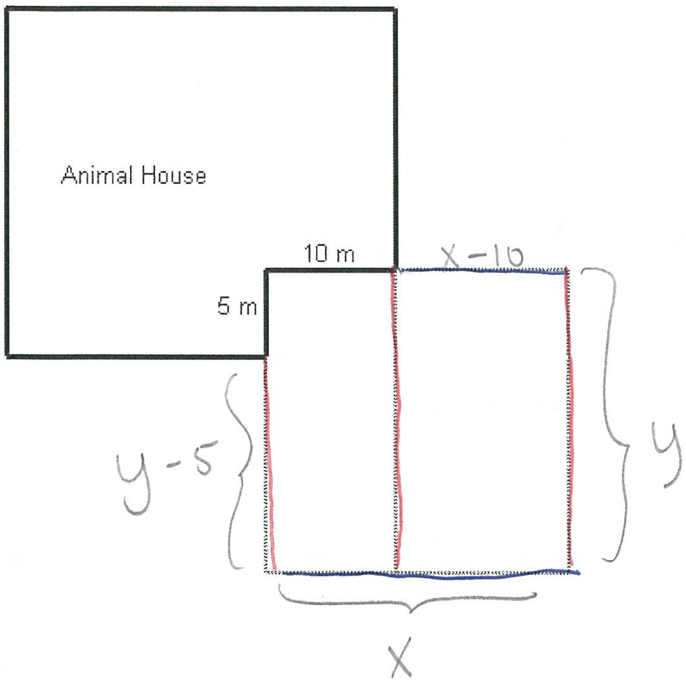
13 ← remainder

ie $\frac{3x^2+1}{x-2} = \underbrace{3x+6}_{} + \frac{13}{x-2}$ goes to 0 as $x \to \pm\infty$

$\boxed{y = 3x + 6 \text{ is the slant asymptote}}$

9. (14 pts). A zoo needs to add a rectangular outdoor pen to an animal house with a corner notch [See figure below]. Fencing will be used to divide the pen into two regions as shown. For the health of the animals, the *total* area of the outdoor pen must be 800 square meters. What dimensions of the pen will minimize the amount of fencing used?

[Note: No fence will be used along the walls of the animal house.]



$$A = xy$$
$$800 = xy$$

Fencing

$$F = x + 2y + x - 10 + y - 5$$
$$F = 2x + 3y - 15$$

Minimize $F = 2x + 3y - 15$ subject to $xy = 800$.

$$y = \frac{800}{x}$$

$$F = 2x + 3\left(\frac{800}{x}\right) - 15$$
$$F = 2x + 2400x^{-1} - 15$$
$$F'(x) = 2 - 2400x^{-2}$$
$$= 2 - \frac{2400}{x^2}$$
$$= \frac{2x^2 - 2400}{x^2}$$

$F' = 0$

$$\frac{2x^2 - 2400}{x^2} = 0$$
$$2x^2 - 2400 = 0$$
$$2x^2 = 2400$$
$$x^2 = 1200$$
$$x = \pm\sqrt{1200}$$

F' DNE

$$x^2 = 0$$
$$x = 0$$

Not physically relevant.

only positive root relevant.

$$\boxed{\begin{aligned} x &= \sqrt{1200} = 20\sqrt{3} \text{ m} \\ y &= \frac{800}{\sqrt{1200}} = \frac{800}{20\sqrt{3}} = \frac{40}{\sqrt{3}} \text{ m} \end{aligned}}$$