Name: Key

Math 151-02, Calculus I – Crawford

Exam 2-A

12 October 2017

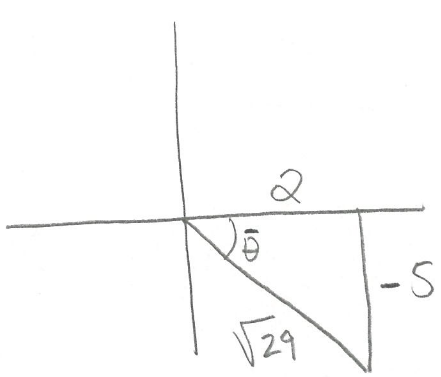
	Score
1	/10
2	/12
3	/12
4	/24
5	/16
6	/10
7	/6
8	/6
9	/6
Total	/100

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- You may use the attached unit circle.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- ***Good luck***!

1. (10 pts). If $\cot\theta = -\frac{2}{5}$ and $\underbrace{\frac{3\pi}{2} \le \theta \le 2\pi}_{Q\,IV}$, use a right triangle to determine $\sin\theta$.

$\cot\theta = -\frac{2}{5} \quad \left(\frac{adj}{opp}\right)$

$$\boxed{\sin\theta = \frac{-5}{\sqrt{29}}}$$



$(2)^2 + (-5)^2 = c^2$

$4 + 25 = c^2$

$29 = c^2$

$c = \pm\sqrt{29}$

2. (12 pts). Find *all* solutions to the following equation.

$2\sin^2 x = \sin x$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0 \qquad 2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\boxed{x = 0 + 2n\pi \qquad x = \pi + 2n\pi}$

for $n \in \mathbb{Z}$

$\boxed{x = \frac{\pi}{6} + 2n\pi \qquad x = \frac{5\pi}{6} + 2n\pi}$

3. (12 pts). Find an equation of the tangent line to $y = \dfrac{3x^2 - 2x}{x^2+1}$ at $x = 1$.

① pt: $y = \dfrac{3(1)^2 - 2(1)}{(1)^2+1} = \dfrac{3-2}{2} = \dfrac{1}{2} \Rightarrow$ pt $(1, \frac{1}{2})$

② Slope: $\dfrac{dy}{dx} = \dfrac{(x^2+1)(6x-2) - (3x^2-2x)(2x)}{(x^2+1)^2}$

$\left.\dfrac{dy}{dx}\right|_{x=1} = \dfrac{(1^2+1)(6(1)-2) - (3(1)^2 - 2(1))(2(1))}{(1^2+1)^2}$

$= \dfrac{(2)(4) - (1)(2)}{(2)^2} = \dfrac{8-2}{4} = \dfrac{6}{4} = \dfrac{3}{2}$

$= m$

$\boxed{y - \frac{1}{2} = \frac{3}{2}(x-1)}$

4. (24 pts). Differentiate the following **[Do not simplify!]**

(a). $s(t) = 3t^4 - 5t - \frac{2}{t^4}$ $= 3t^4 - 5t - 2t^{-4}$

$$\boxed{s'(t) = 12t^3 - 5 + 8t^{-5}}$$

(b). $y = x^2 \sec x$

$$\boxed{y' = x^2 \cdot \sec x \tan x + (\sec x) \cdot 2x}$$

$= x^2 \sec x \tan x + 2x \sec x$

(c). $f(x) = \cos(\sin(4x^2))$

$f'(x) = -\sin(\sin(4x^2)) \cdot \frac{d}{dx}[\sin(4x^2)]$ CR

$= -\sin(\sin(4x^2)) \cdot \cos(4x^2) \cdot \frac{d}{dx}[4x^2]$

$= \boxed{-\sin(\sin(4x^2)) \cdot \cos(4x^2) \cdot 8x}$

5. (16 pts). Given the curve $x + x^2y = y^3$, use implicit differentiation to

(a). Find y'.

$$\frac{d}{dx}\left[x + x^2y\right] = \frac{d}{dx}\left[y^3\right]$$

$$1 + x^2 \cdot y' + y \cdot 2x = 3y^2 \cdot y'$$

$$x^2y' - 3y^2y' = -1 - 2xy$$

$$y'(x^2 - 3y^2) = -1 - 2xy$$

$$\boxed{y' = \frac{-1-2xy}{x^2-3y^2}}$$

(b). Find y'' in terms of x and y only. [You do not need to simplify.]

$$y'' = \frac{d}{dx}\left[\frac{-1-2xy}{x^2-3y^2}\right]$$

$$= \frac{(x^2-3y^2)\cdot \frac{d}{dx}\left[-1-2xy\right] - (-1-2xy)\frac{d}{dx}\left[x^2-3y^2\right]}{(x^2-3y^2)^2}$$

$$= \frac{(x^2-3y^2)\cdot\left(-2xy' + y(-2)\right) - (-1-2xy)(2x - 6yy')}{(x-3y^2)^2}$$

$$= \boxed{\frac{(x^2-3y^2)\cdot\left(-2x\cdot\frac{-1-2xy}{x^2-3y^2} - 2y\right) - (-1-2xy)\left(2x - 6y\cdot\frac{-1-2xy}{x^2-3y^2}\right)}{(x^2-3y^2)^2}}$$

6. (10 pts). Given $f(x) = \sqrt{3 + x^2}$ $= (3+x^2)^{1/2}$

(a). Find the differential dy.

$dy = f'(x)\,dx$

$f'(x) = \frac{1}{\cancel{2}}(3+x^2)^{-1/2} \cdot \cancel{2}x$

$= \frac{x}{(3+x)^{1/2}}$

$\boxed{dy = \frac{x}{\sqrt{3+x^2}}\,dx}$

(b). Evaluate dy for $x = 1$ and $dx = -0.1$. [Simplify your answer.]

$dy = \frac{1}{\sqrt{3+1^2}} \cdot (-0.1) = \frac{-0.1}{\sqrt{4}} = -\frac{0.1}{2} = \boxed{-0.05}$

7. (6 pts). If the mass (in g) of a thin metal rod is given by $m(x) = 9 + x^2$,

find the linear density ρ when $x = 3$ m. [Include units in your answer.]

$\rho = \frac{dm}{dx} = 2x$

$\rho\big|_{x=3} = 2(3) = \boxed{6\ g/m}$

8. (6 pts). Boyle's Law states that when a sample of gas is compressed at a constant temperature, the product of the pressure and the volume remains constant:

$PV = C$ Find the rate of change of volume with respect to pressure.

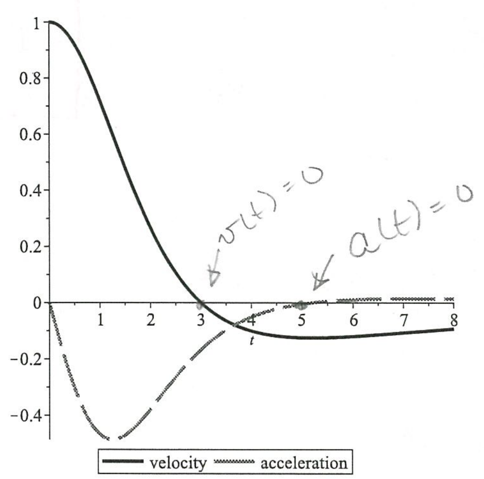
Find $\frac{dV}{dP}$

$V = \frac{C}{P}$

$V = CP^{-1}$

$$\boxed{\frac{dV}{dP} = -CP^{-2} = \frac{-C}{P^2}}$$

9. (6 pts). The graph below shows the velocity $v(t)$ and acceleration $a(t)$ of a particle at time t.



(a). Determine the time(s) when the particle is at rest.

ie When is $v(t) = 0$?

Ans: $\boxed{t = 3}$

(b). Determine the time interval(s) when the particle is speeding up.

Both $v(t)$ and $a(t)$ have the same sign

On $3 < t < 5$ both $v(t)$ and $a(t)$ are negative $\Rightarrow$ speeding up.

Ans: $\boxed{3 < t < 5}$