Name: Key

Math 151-01, Calculus I – Crawford

Exam 2-B/C

12 October 2017

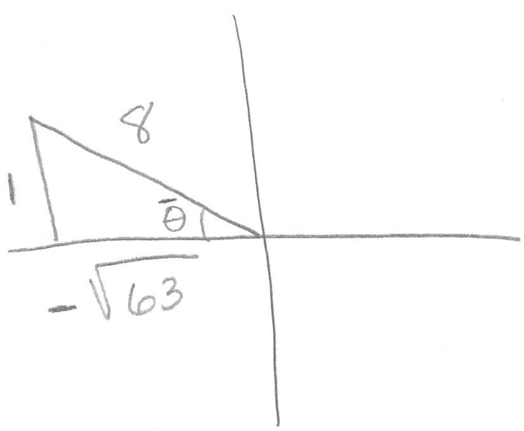
	Score
1	/10
2	/12
3	/12
4	/24
5	/16
6	/12
7	/16
Total	/100

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- You may use the attached unit circle.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- ***Good luck***!

Version C #2

1. (10 pts). If $\sin\theta = \frac{1}{8}$ and $\underbrace{\frac{\pi}{2} \le \theta \le \pi}_{QII}$, use a right triangle to determine $\tan\theta$.

$\sin\theta = \frac{1}{8}$ $\left(\frac{opp}{hyp}\right)$



$$\boxed{\tan\theta = -\frac{1}{\sqrt{63}}}$$

$(1)^2 + b^2 = 8^2$

$1 + b^2 = 64$

$b^2 = 63$ $\Rightarrow$ $b = \pm\sqrt{63}$

2. (12 pts). Find *all* solutions to the following equation. Version C #1

$\cos(3x) = \frac{1}{2}$

$$3x = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad 3x = \frac{5\pi}{3} + 2n\pi$$

$$\Rightarrow \boxed{x = \frac{\pi}{9} + \frac{2n\pi}{3} \quad \text{or} \quad x = \frac{5\pi}{9} + \frac{2n\pi}{3} \quad n \text{ any integer}}$$

3. (12 pts). Find an equation of the tangent line to $y = \dfrac{2x^3 - 3x^2}{2 - x^2}$ at $x = 1$.

① pt: $y = \dfrac{2(1)^3 - 3(1)^2}{2 - (1)^2} = \dfrac{2-3}{2-1} = \dfrac{-1}{1} = -1$

$\Rightarrow$ pt $(1, -1)$

② Slope: $\dfrac{dy}{dx} = \dfrac{(2-x^2)(6x^2-6x) - (2x^3-3x^2)(-2x)}{(2-x^2)^2}$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{(2-1^2)(6(1)^2-6(1)) - (2(1)^3-3(1)^2)(-2(1))}{(2-1^2)^2}$$

$$= \frac{(1)(0) - (-1)(-2)}{1^2} = \frac{-2}{1} = -2 = m$$

$$\boxed{y + 1 = -2(x-1)}$$

4. (24 pts). Differentiate the following **[Do not simplify!]**

(a). $y = 4x^5 - 3x - \pi^2$

$$\boxed{y' = 20x^4 - 3}$$

(b). $y = x^2\sqrt{x} + \sec x$ $= x^2 \cdot x^{1/2} + \sec x = x^{5/2} + \sec x$

$$\boxed{y' = \frac{5}{2}x^{3/2} + \sec x \tan x}$$

(c). $f(x) = \sin(\cos(ax^3))$ for a constant a.

$$f'(x) = \cos(\cos(ax^3)) \cdot \overset{\text{CR}}{\frac{d}{dx}}[\cos(ax^3)]$$

$$= \cos(\cos(ax^3)) \cdot (-\sin(ax^3)) \cdot \overset{\text{CR}}{\frac{d}{dx}}[ax^3]$$

$$= \boxed{\cos(\cos(ax^3)) \cdot (-\sin(ax^3)) \cdot 3ax^2}$$

5. (16 pts). Given the curve $y + 5 = xy^3$, use implicit differentiation to

(a). Find y'.

$$\frac{d}{dx}[y+5] = \frac{d}{dx}[xy^3]$$

$$y' + 0 = x \cdot 3y^2 y' + y^3 \cdot 1$$

$$y' - 3xy^2 y' = y^3$$

$$y'(1-3xy^2) = y^3$$

$$\boxed{y' = \frac{y^3}{1-3xy^2}}$$

(b). Find y'' in terms of x and y only. [You do not need to simplify.]

$$y'' = \frac{d}{dx}\left[\frac{y^3}{1-3xy^2}\right]$$

$$= \frac{(1-3xy^2)\cdot 3y^2y' - y^3\cdot(-3x\cdot 2yy' + y^2(-3))}{(1-3xy^2)^2}$$

$$= \boxed{\frac{(1-3xy^2)\cdot 3y^2\frac{y^3}{1-3xy^2} - y\left(-6xy\cdot\frac{y^3}{1-3xy^2} - 3y^2\right)}{(1-3xy^2)^2}}$$

6. (12 pts). Given $f(x) = \frac{1}{x} = x^{-1}$

(a). Find the linearization $L(x)$ at $x = 10$.

① pt. $f(10) = \frac{1}{10} \Rightarrow \left(10, \frac{1}{10}\right)$

② slope: $f'(x) = -x^{-2} = -\frac{1}{x^2}$

$f'(10) = -\frac{1}{(10)^2} = -\frac{1}{100} = m$

$y - \frac{1}{10} = -\frac{1}{100}(x-10)$

$$\boxed{L(x) = \frac{1}{10} - \frac{1}{100}(x-10)}$$

(b). Use the linearization from part (a) to approximate $\frac{1}{10.1}$. i.e. Use $L(x)$ to approximate $f(10.1)$.

[You do **not** need to simplify the approximation in part (b).... Seriously, don't simplify it.]

$$\frac{1}{10.1} = f(10.1) \approx L(10.1) = \boxed{\frac{1}{10} - \frac{1}{100}(10.1-10)}$$

Not Simplified - Full Credit

$= \frac{1}{10} - \frac{1}{100}(0.1)$

$= \frac{1}{10} - \frac{1}{100}\left(\frac{1}{10}\right)$

$= \frac{1}{10} - \frac{1}{1000}$

$= \frac{100}{1000} - \frac{1}{1000}$

$= \frac{99}{1000} = .099$

7. (16 pts). The height at time t (in seconds) of an object shot upward is given by $s(t) = -16t^2 + 64t + 8$ in feet.

(a). Find the velocity and acceleration at time t.

$$\boxed{v(t) = -32t + 64 \quad ft/s}$$

$$\boxed{a(t) = -32 \quad ft/s^2}$$

(b). When is the object at rest?

ie When does $v(t) = 0$?

$$-32t + 64 = 0$$
$$64 = 32t$$
$$\boxed{t = 2 \text{ s}}$$

(c). Is the object speeding up or slowing down when $t = 1$? [Justify your answer.]

$$v(1) = -32(1) + 64 = -32 + 64 = 32 > 0$$

$$a(1) = -32 < 0$$

opposite signs

$\Rightarrow$ $\boxed{\text{slowing down}}$