

1. Solve the following inequalities. Write your solution in interval notation and sketch it on the numberline.

(a). $2x^2 + x \geq 3$ $(-\infty, -\frac{3}{2}] \cup [1, \infty)$

(b). $\frac{(x-3)^2(x+1)}{x+4} \geq 0$ $(-\infty, -4) \cup [-1, \infty)$

2. Section 1.1, #10

No, it is not a function. It fails the VLT.

3. Given $f(x) = \begin{cases} -1, & x \leq -1 \\ x, & -1 < x \leq 2 \\ x^2 + 1, & x > 2 \end{cases}$

(a). Find the domain and sketch the function.

domain: All real numbers

(b). Find $f(-2)$, $f(2)$, and $f(4)$.

$f(-2) = -1$, $f(2) = 2$, $f(4) = 17$

4. Find the domain of $f(x) = \frac{1}{\sqrt{x^2 + x}}$.

$(-\infty, -1) \cup (0, \infty)$

5. Section 1.4 #7

6. Section 1.5 #7, 9, 17

7. Evaluate the following limits, if they exist (clearly indicate $+\infty$ or $-\infty$ in the case of an infinite limit). If the limit does not exist, **explain the reason why**.

(a). $\lim_{x \rightarrow 0} \frac{x-3}{x(x+4)}$ DNE (one-sided limits are different)

(d). $\lim_{x \rightarrow 0} \frac{x-3}{x^2(x+4)} = -\infty$

(b). $\lim_{x \rightarrow -4} \frac{x^2 + 2x - 8}{x + 4} = -6$

(e). $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{1}{2\sqrt{x}}$

(c). $\lim_{x \rightarrow 0} \frac{\sqrt{3x^2 + 4}}{x - 4} = -\frac{1}{2}$

(f). $\lim_{x \rightarrow 1} f(x)$, where $f(x) = \begin{cases} 3, & x \leq 1 \\ 1, & x > 1 \end{cases}$
DNE (one-sided limits are different)

8. Given that $\lim_{x \rightarrow 1} f(x) = 2$, $\lim_{x \rightarrow 1} g(x) = 4$, $\lim_{x \rightarrow 1} h(x) = 0$, find the following limits if they exist.

(a). $\lim_{x \rightarrow 1} f(x) - g(x) = -2$

(b). $\lim_{x \rightarrow 1} f(x) \cdot g(x) = 8$

(c). $\lim_{x \rightarrow 1} h(x)/g(x) = 0$

(d). $\lim_{x \rightarrow 1} g(x)/h(x)$

DNE (One-sided limits are $+\infty$ or $-\infty$, but not enough info to determine which one or if they agree.)

9. For each of the following functions,

(i) find all of the x -values, if any, where $g(x)$ is discontinuous and

(ii) indicate whether it is a removable, infinite, or jump discontinuity.

(a). $g(x) = \frac{x^2 - x - 6}{x(x^2 - 9)}$

infinite at $x = 0$, infinite $x = -3$, removable at $x = 3$

(b). $f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ 1 - x, & x > 1 \end{cases}$

Continuous everywhere

10. Given the function $f(x) = x^3 - 2x^2 + 8x - 1$, use the Intermediate Value Theorem to show that there is a number c where $0 < c < 2$, such that $f(c) = 6$. $f(0) = -1$ and $f(2) = 15$. Since $-1 \leq 6 \leq 15$ AND f is continuous, then the IVT guarantees $f(x)$ must pass through $y = 6$ for some value of $x = c$ in the interval $(0, 2)$.

11. Suppose $f(1) = 3$, $f'(1) = -2$, $f(5) = 8$, and $f'(5) = 15$. Let P be the point on the graph $y = f(x)$ where $x = 1$. Let Q be the point on the graph of $y = f(x)$ where $x = 5$.

(a). Find the equation of the secant line PQ .

$$y - 3 = \frac{5}{4}(x - 1)$$

(b). Find the equation of the tangent line to $y = f(x)$ at P .

$$y - 3 = -2(x - 1)$$

12. Section 2.1 #49

$T'(8)$ represents the rate at which the temperature is changing at 8am. $T'(8) \approx 3.75^\circ \text{ F/hour}$.

13. Given $f(x) = x^2 - x$, use the limit definition $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ to find the slope of the tangent line to the curve at $x = 2$. Use your result to write an equation of the tangent line at $x = 2$.

$$m = 3, y - 2 = 3(x - 2)$$

14. Suppose the position of a particle at time t seconds is given by $s(t) = \sqrt{t}$ meters. Use the limit definition to find the velocity of the particle at time $t = 5$.

$$v(5) = \frac{1}{2\sqrt{5}}$$

15. Use the limit definition of the derivative to find $f'(x)$ for the following:

[You must use the limit definition.]

(a). $f(x) = \frac{1}{x^2}$ Simplify your answer.

$$f'(x) = -\frac{2}{x^3}$$

(b). $f(x) = 2x^2 + 3x$

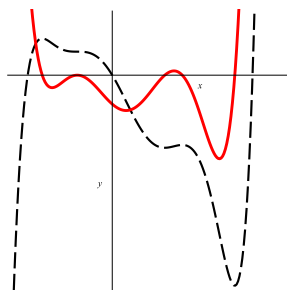
$$f'(x) = 4x + 3$$

16. Find the equation of the tangent line to the curve $y = 2x^2 + 3x$ at the point $(1, 5)$.

$$y - 5 = 7(x - 1)$$

[Hint: See (b) in previous problem.]

17. Both the function f and its derivative f' are plotted on the same set of axes below. Which curve represents the function and which curve represents the derivative? Justify your answer. dashed (black) is $f(x)$ solid (red) is $f'(x)$ Everywhere the dashed (black) curve has a horizontal tangent line (slope = 0), the solid (red) curve goes through 0.



18. Section 2.2 #3, 15, 39.

19. Use the limit definition $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ to show that $f(x) = |x - 5|$ is not differentiable at $x = 5$. Simplify the limit and use the definition of $|x|$ to show that the one-sided limits are different.