Name: Key

Math 151-02, Calculus I – Crawford

Version B

Differences Noted by each problem.

Exam 1-A

19 September 2017

	Score
1	/8
2	/12
3	/8
4	/24
5	/16
6	/16
7	/6
8	/6
9	/6
Total	/100

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- Problems #1 & 2 will be used to determine extra-credit for Homework Check 1.
- ***Good luck***!

Version B #2

1. (8 pts). Find the domain of $f(x) = \dfrac{\sqrt{3-2x}}{x}$.

$3 - 2x \geq 0$ AND $x \neq 0$

$3 \geq 2x$

$\frac{3}{2} \geq x$

ie $\boxed{x \leq \frac{3}{2} \text{ AND } x \neq 0}$

Version B#3

2. (12 pts). Solve the following inequality for x.

$x^3 - 3x^2 - 10x \leq 0$

$x(x^2 - 3x - 10) \leq 0$

$x(x-5)(x+2) \leq 0$

$x = 0, 5, -2$

$\boxed{(-\infty, -2] \cup [0, 5]}$

$-$ $+$ $-$ $+$

$(-)(-)(-)$ | $(-)(-)(+)$ | $(+)(-)(+)$ | $(+)(+)(+)$ x

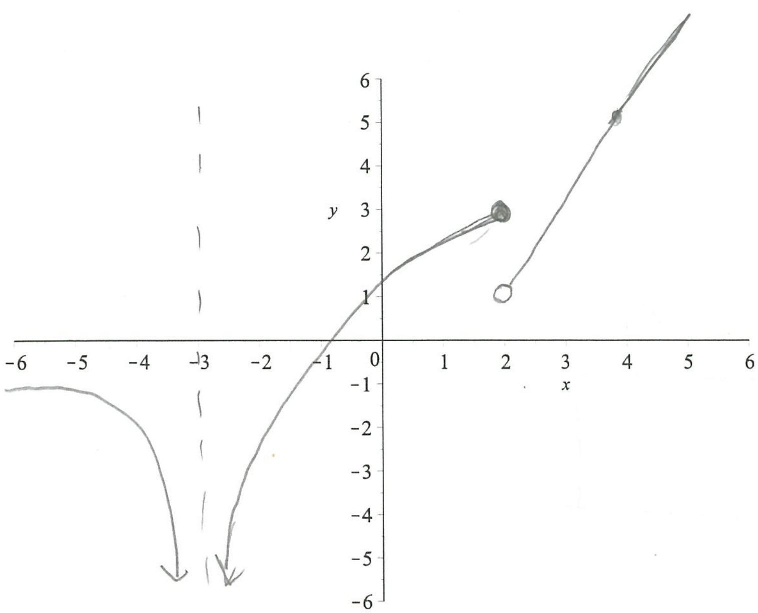
-2 0 5

$x = -10$ $x = -1$ $x = 2$ $x = 10$

Version B #1

3. (8 pts). Sketch a function that has all of the following properties.

$\lim_{x \to 2^-} f(x) = 3$, $\lim_{x \to 2^+} f(x) = 1$, $f(2) = 3$, $\lim_{x \to -3} f(x) = -\infty$, $\lim_{x \to 4} f(x) = 5$ (oops)



4. (24 pts). Evaluate the following limits, if they exist. Clearly indicate $+\infty$ or $-\infty$ in the case of an infinite limit. If the limit does not exist, **clearly explain the reason why**.

(a). $\lim_{x\to -3} \frac{x^2+3x}{x^2+2x-3} = \lim_{X\to -3} \frac{X\cancel{(X+3)}}{\cancel{(X+3)}(X-1)} = \lim_{X\to -3} \frac{X}{X-1}$

$\frac{(-3)^2+3(-3)}{(-3)^2+2(-3)-3}$

$\frac{9-9}{9-6-3}$

$\frac{0}{0}$ Indeterminate Form

$= \frac{-3}{-3-1} = \frac{-3}{-4} = \boxed{\frac{3}{4}}$

(b). $\lim_{x\to 1} \frac{2x-1}{x-1}$

$\frac{2-1}{1-1}$

$\frac{1}{0}$

Infinite Limit

(Check one-sided)

$\lim_{X\to 1^-} \frac{2X-1}{X-1} = -\infty$

eg 0.9 $\frac{1.8-1}{0.9-1} \to \frac{(+)}{(-)} \to (-)$

$\lim_{X\to 1^+} \frac{2X-1}{X-1} = +\infty$

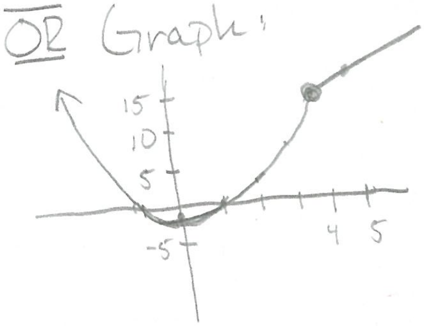
eg 1.1 $\frac{2.2-1}{1.1-1} \to \frac{(+)}{(+)} \to (+)$

The one-sided limits are different.

$\therefore \lim_{X\to 1} \frac{2X-1}{X-1}$ DNE

(c). $\lim_{x\to 4} g(x)$ where $g(x) = \begin{cases} x^2-1, & x \le 4 \\ 3x+3, & x > 4 \end{cases}$

OR Graph:



$\lim_{\underbrace{X\to 4^+}_{X>4}} g(x) = 3(4)+3 = 15$

one-sided limits are equal

$\lim_{\underbrace{X\to 4^-}_{X<4}} g(x) = (4)^2-1 = 16-1 = 15$

$\therefore$

$\boxed{\lim_{X\to 4} g(x) = 15}$

5. (16 pts). The position of a particle at time t seconds is given by $s(t) = \sqrt{2t}$ cm.

(a). Find the average velocity of the particle over the time interval $[2, 8]$. [Include units in your answer.]

$S(2) = \sqrt{2\cdot 2} = \sqrt{4} = 2$

$S(8) = \sqrt{2\cdot 8} = \sqrt{16} = 4$

$v_{ave} = \dfrac{S(8) - S(2)}{8-2} = \dfrac{4-2}{6} = \dfrac{2}{6} = \boxed{\dfrac{1}{3}\text{ cm/s}}$

(b). **Use the limit definition** $v(a) = \lim\limits_{t \to a} \dfrac{s(t) - s(a)}{t - a}$ to find the instantaneous velocity when $t = 8$.
[Include units in your answer.] ***You must use the limit definition and you must show all of your work.***

$$v(8) = \lim_{t \to 8} \frac{S(t) - S(8)}{t-8}$$

$$= \lim_{t \to 8} \frac{\sqrt{2t} - 4}{t-8} \cdot \frac{\sqrt{2t}+4}{\sqrt{2t}+4}$$

$$= \lim_{t \to 8} \frac{(\sqrt{2t})^2 - (4)^2}{(t-8)(\sqrt{2t}+4)}$$

$$= \lim_{t \to 8} \frac{2t - 16}{(t-8)(\sqrt{2t}+4)}$$

$$= \lim_{t \to 8} \frac{2\cancel{(t-8)}}{\cancel{(t-8)}(\sqrt{2t}+4)}$$

$$= \lim_{t \to 8} \frac{2}{\sqrt{2t}+4} = \frac{2}{\sqrt{2\cdot 8}+4} = \frac{2}{4+4} = \frac{2}{8} = \boxed{\frac{1}{4}\text{ cm/s}}$$

[Note: $s'(t) = v(t) = \frac{1}{\sqrt{2t}}$, if you want to check your answer.]

Check: $S'(8) = v(8) = \dfrac{1}{\sqrt{2\cdot 8}} = \dfrac{1}{\sqrt{16}} = \dfrac{1}{4}$ ✓

6. (16 pts). Let $f(x) = 1 - 3x^2$.

(a). ***Use the limit definition*** $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ to show that the derivative is $f'(x) = -6x$.

You must use the limit definition and you must show all of your work.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{1-3(x+h)^2-(1-3x^2)}{h}$$

$$= \lim_{h\to 0} \frac{1-3(x+h)^2-1+3x^2}{h}$$

$$= \lim_{h\to 0} \frac{1-3(x^2+2xh+h^2)-1+3x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{1}-\cancel{3x^2}-6xh-3h^2-\cancel{1}+\cancel{3x^2}}{h}$$

$$= \lim_{h\to 0} \frac{-6xh-3h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-6x-3h)}{\cancel{h}}$$

$$= \lim_{h\to 0} (-6x-3h)$$

$$= \boxed{-6x}$$ ✓

(b). Find an equation of the tangent line to the graph at $x = 2$.

① pt : $f(2) = 1-3(2)^2 = 1-3(4) = 1-12 = -11$

ie pt $(2, -11)$

② Slope : $m = f'(2) = -6(2) = -12$

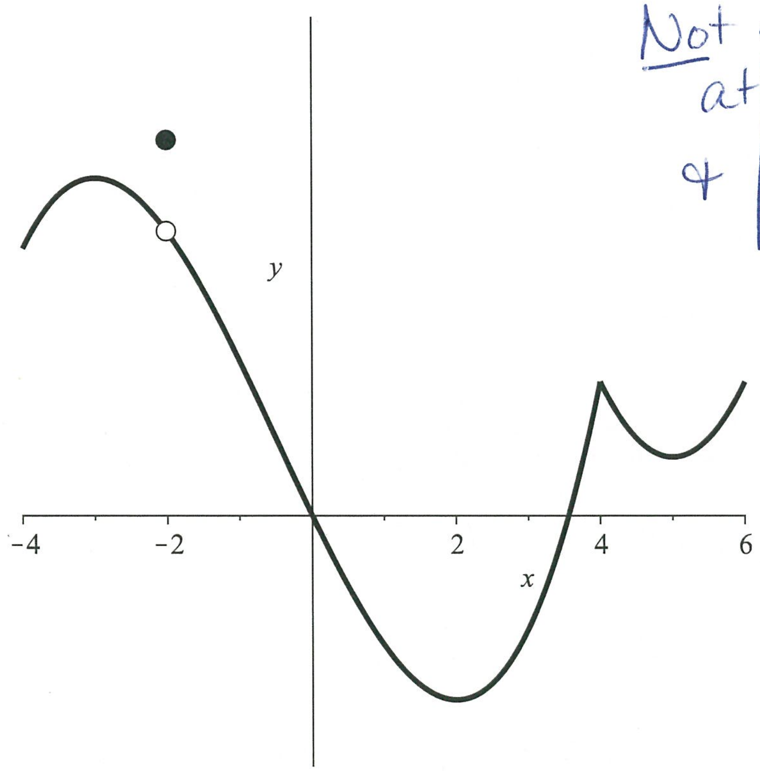
$$\boxed{y + 11 = -12(x-2)}$$

7. (6 pts). Write down a function $f(x)$ whose graph has an infinite discontinuity at $x = 3$ and a removable discontinuity at $x = 6$. [You must write down a function $f(x)$. A graph of the function is not acceptable.]

$$\boxed{f(x) = \frac{(x-6)}{(x-6)(x-3)}}$$

There are other possibilities.

8. (6 pts). The graph of f is given below. For which values of x is f not differentiable?



Not differentiable at $\boxed{x = -2}$ & $\boxed{x = 4}$

9. (6 pts). *True or False.* Clearly indicate whether the following statements are true or false. Expanations are not required.

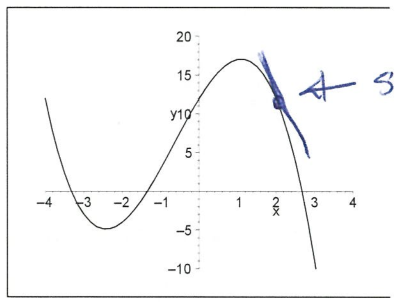
T F If $f(x) = \dfrac{12}{x}$ then the Intermediate Value Theorem guarantees that $f(x)$ will go through $y = 1$ for some value of x in the interval $(-2, 1)$.

$f(x) = \frac{12}{x}$ is discontinuous at $x = 0$

ie f is not cont. on $(-2,1)$, so the conditions of the theorem are not met.

$\Rightarrow$ No guarantee

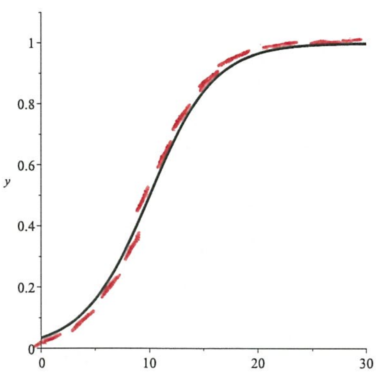
T F If the graph of a function $y = f(x)$ is given below, then the derivative $f'(2) > 0$.



slope of tangent line is neg.

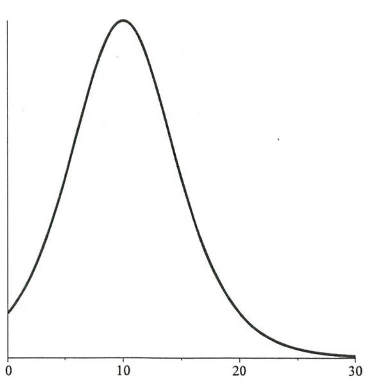
$\Rightarrow f'(2) < 0$

T F Given the graph of $y = f(x)$ below left, its derivative is given by the graph below it.



$y = f(x)$

Slopes of tangent lines (ie derivative) starts off shallow (small) & pos., gets steeper (larger) & still pos., then gets shallower (smaller) & still pos.



Is this the derivative $f'(x)$?

Version B T/F in a different order.