Name: Key

Math 151-01, Calculus 1 – Crawford

NOTE: VersionD is noted by each problem.

Exam $\boxed{1\text{-C}}$

19 September 2017

Score	
1	/8
2	/12
3	/8
4	/24
5	/16
6	/16
7	/6
8	/6
9	/6
Total	/100

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- Problems #1 & 2 will be used to determine extra-credit for Homework Check 1.
- ***Good luck***!

VersionD #2

1. (8 pts). Find the domain of $f(x) = \dfrac{\sqrt{4x-3}}{x-5}$.

$x - 5 \neq 0$ AND $4x - 3 \geq 0$

$\boxed{x \neq 5}$ $\quad 4x \geq 3$

$\boxed{x \geq \frac{3}{4}}$

2. (12 pts). Solve the following inequality for x. Version D #1

$x^3 - 10x^2 + 25x \geq 0$

$X(X^2-10X+25) \geq 0$

$X(X-5)^2 \geq 0$

$X = 0, 5$

$-$ $+$ $+$

$(-)(+)$ | $(+)(+)$ | $(+)(+)$ X

0   5

$X=-10$ $X=1$ $X=10$

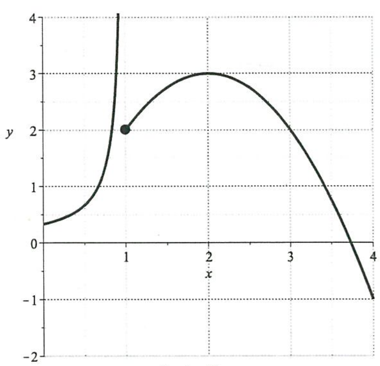
$\boxed{[0, \infty)}$

OR

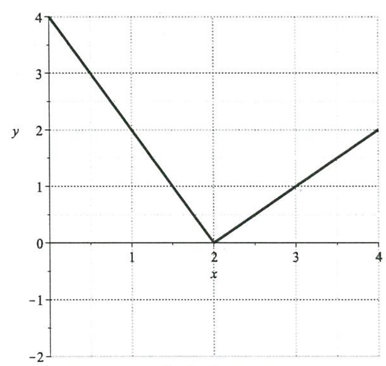
0

OR $X \geq 0$

3. (8 pts). Given the graphs of f and g below, determine the following limits. Clearly indicate $+\infty$ or $-\infty$ in the case of an infinite limit.



Graph of f



Graph of g

(a). $\lim_{x \to 1^-} f(x) = \boxed{+\infty}$

Version D(c)

(b). $\lim_{x \to 3} (f(x) + g(x))$

$= 2 + 1$

$= \boxed{3}$

Version D (b)

(c). $\lim_{x \to 1^+} f(x) \cdot g(x)$

$= 2 \cdot 2$

$= \boxed{4}$

(d). $\lim_{x \to 2} \dfrac{f(x)}{g(x)}$ Multiple Choice for part (d) only:

(A). 0   (B). 2   (C). $-\infty$ **(D). ∞**   (E). DNE

same

$\frac{3}{0}$ ← infinite limit ⇒ check one-sided

$\lim_{X \to 2^-} \frac{f(x)}{g(x)} = +\infty$ eg 1.9 $\frac{(+)}{(+)}$

$\lim_{X \to 2^+} \frac{f(x)}{g(x)} = +\infty$ eg 2.1 $\frac{(+)}{(+)}$

4. (24 pts). Evaluate the following limits, if they exist. Clearly indicate $+\infty$ or $-\infty$ in the case of an infinite limit. If the limit does not exist, **clearly explain the reason why.**

Version D (b) **(a).** $\displaystyle\lim_{x\to 4}\frac{9-2x}{x-4}$

$\frac{9-8}{4-4}=\frac{1}{0}$

Infinite Limit

$\Rightarrow$ Check one sided

$\displaystyle\lim_{x\to 4^-}\frac{9-2x}{x-4}=-\infty$

eg 3.9 $\frac{9-7.8}{3.9-4}\to\frac{(+)}{(-)}\to(-)$

$\displaystyle\lim_{x\to 4^+}\frac{9-2x}{x-4}=+\infty$

eg 4.1 $\frac{9-8.2}{4.1-4}\to\frac{(+)}{(+)}\to(+)$

one-sided limits are different. $\therefore \displaystyle\lim_{x\to 4}\frac{9-2x}{x-4}$ DNE

Version D (a) **(b).** $\displaystyle\lim_{x\to -2}\frac{x^2+2x}{x^2+6x+8} = \lim_{x\to -2}\frac{x\cancel{(x+2)}}{\cancel{(x+2)}(x+4)} = \lim_{x\to -2}\frac{x}{x+4}$

$\frac{(-2)^2+2(-2)}{(-2)^2+6(-2)+8}$

$\frac{4-4}{4-12+8}=\frac{0}{0}$

Indeterminate Form.

$=\frac{-2}{-2+4}=\frac{-2}{2}=\boxed{-1}$

(c). $\displaystyle\lim_{x\to 8}\frac{\sqrt{2x}-4}{x-8}\cdot\frac{\sqrt{2x}+4}{\sqrt{2x}+4} = \lim_{x\to 8}\frac{(\sqrt{2x})^2-(4)^2}{(x-8)(\sqrt{2x}+4)}$

$\frac{\sqrt{2\cdot 8}-4}{8-8}$

$\frac{\sqrt{16}-4}{0}$

$\frac{4-4}{0}$

$\frac{0}{0}$

Ind. Form.

$=\displaystyle\lim_{x\to 8}\frac{2x-16}{(x-8)(\sqrt{2x}+4)}$

$=\displaystyle\lim_{x\to 8}\frac{2\cancel{(x-8)}}{\cancel{(x-8)}(\sqrt{2x}+4)}$

$=\displaystyle\lim_{x\to 8}\frac{2}{\sqrt{2x}+4}$

$=\frac{2}{\sqrt{2\cdot 8}+4}$

$=\frac{2}{4+4}$

$=\frac{2}{8}$

$=\boxed{\frac{1}{4}}$

5. (16 pts). Let $f(x) = 3 - 2x^2$.

(a). ***Use the limit definition*** $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ to show that the derivative is $\boxed{f'(x) = -4x.}$

You must use the limit definition and you must show all of your work.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{3-2(x+h)^2 - (3-2x^2)}{h}$$

$$= \lim_{h\to 0} \frac{3-2(x^2+2xh+h^2) - 3+2x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3} - \cancel{2x^2} - 4xh - 2h^2 - \cancel{3} + \cancel{2x^2}}{h}$$

$$= \lim_{h\to 0} \frac{-4xh - 2h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-4x-2h)}{\cancel{h}} = \lim_{h\to 0} (-4x-2h) = \boxed{-4x} \checkmark = f'(x)$$

(b). Find an equation of the tangent line to the graph at $x = 2$.

① Pt. $f(2) = 3-2(2)^2 = 3-8 = -5$

pt $(2,-5)$

② Slope: $m = f'(2) = -4(2) = -8$

$$\boxed{y + 5 = -8(x-2)}$$

6. (16 pts). The position of a particle at time t seconds is given by $s(t) = \dfrac{1}{2t}$ cm.

(a). Find the average velocity of the particle over the time interval $[1, 2]$. [Simplify your answer and include units.]

$S(1) = \frac{1}{2(1)} = \frac{1}{2}$

$S(2) = \frac{1}{2(2)} = \frac{1}{4}$

$v_{ave} = \dfrac{S(2) - S(1)}{2-1} = \dfrac{\Delta S}{\Delta t}$

$= \dfrac{\frac{1}{4} - \frac{1}{2}}{1} = \frac{1}{4} - \frac{2}{4} = \boxed{-\frac{1}{4}\text{ cm/s}}$

(b). **Use the limit definition** $v(a) = \lim\limits_{t \to a} \dfrac{s(t) - s(a)}{t - a}$ to find the instantaneous velocity when $t = 2$.
[Include units in your answer.] *You must use the limit definition and you must show all of your work.*

$v(2) = \lim\limits_{t\to 2} \dfrac{S(t) - S(2)}{t-2}$

$= \lim\limits_{t\to 2} \dfrac{\frac{1}{2t} - \frac{1}{4}}{t-2}$

cross multiplied

LCD: 4t

OR $\dfrac{\frac{1}{2t}\cdot\frac{2}{2} - \frac{1}{4}\cdot\frac{t}{t}}{t-2} = \dfrac{\left(\frac{2-t}{4t}\right)}{t-2}$ etc

$= \lim\limits_{t\to 2} \dfrac{\left(\frac{4-2t}{2t\cdot 4}\right)}{t-2}$

$= \lim\limits_{t\to 2} \dfrac{4-2t}{8t}\cdot\dfrac{1}{t-2}$

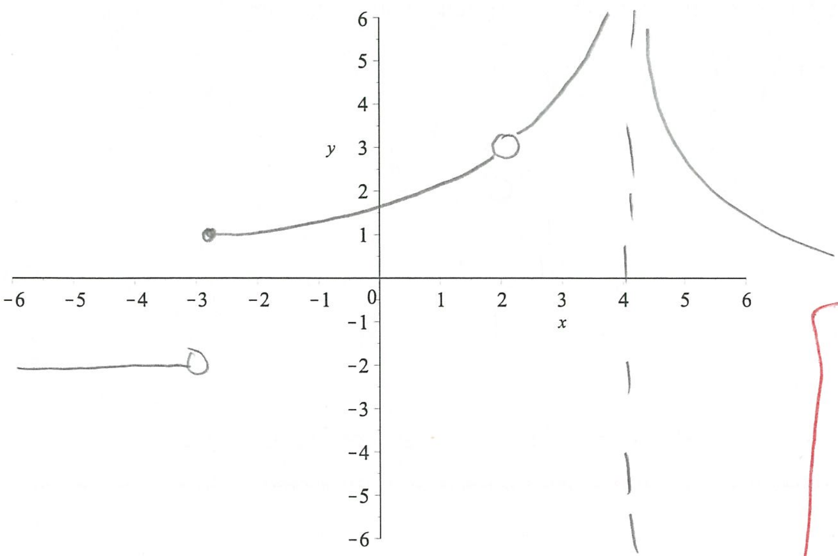
$= \lim\limits_{t\to 2} \dfrac{-2\cancel{(t-2)}}{8t}\cdot\dfrac{1}{\cancel{(t-2)}}$

$\lim\limits_{t\to 2} \dfrac{-2}{8t} = \dfrac{-2}{8(2)}$

$= \boxed{-\frac{1}{8}\text{ cm/s}}$

[Note: $s'(t) = v(t) = -\frac{1}{2t^2}$, if you want to check your answer.] Check: $S'(2) = \dfrac{-1}{2(2)^2} = -\frac{1}{8}$ ✓

7. (6 pts). Sketch a function $f(x)$that has an infinite discontinuity at $x = 4$, a removable discontinuity at $x = 2$, and a jump discontinuity at $x = -3$.



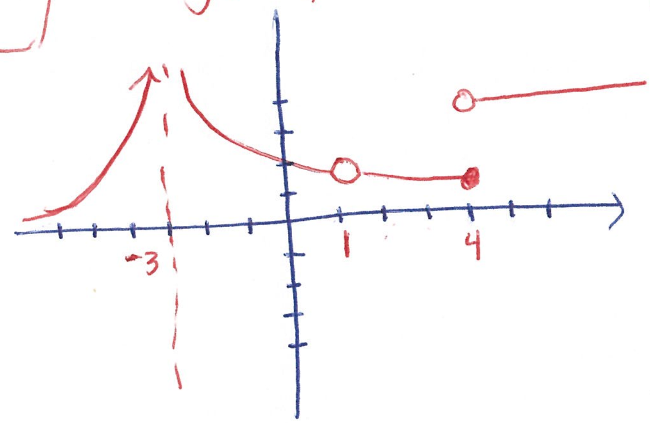
Many Different Possible Functions

Version D

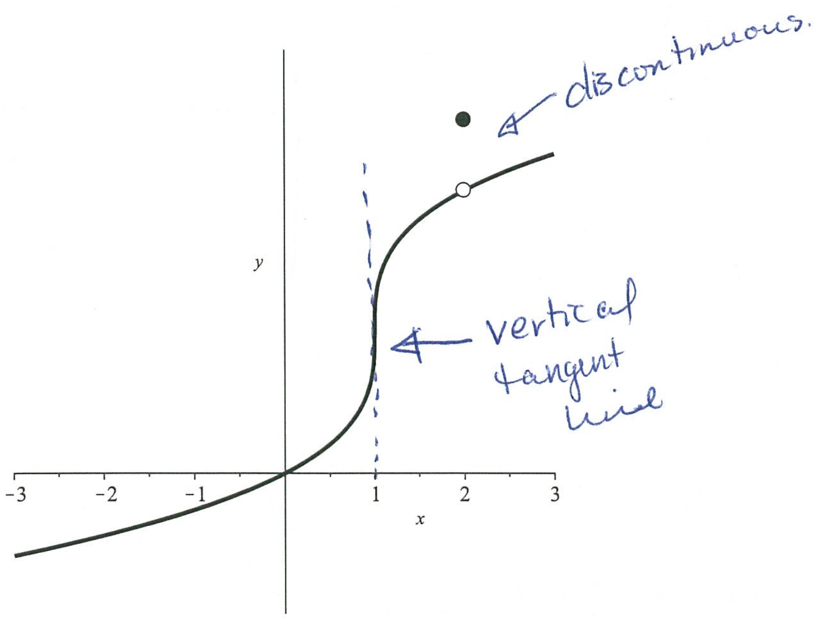
infinite discont. @ $X=-3$

removable discont. @ $X=1$

jump discont @ $X=4$



8. (6 pts). The graph of f is given below. For which values of x is f not differentiable?



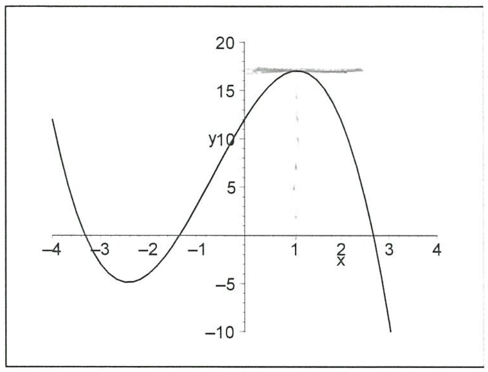
Not differentiable at $X=1$ and $X=2$

9. (6 pts). *True or False.* Clearly indicate whether the following statements are true or false. Explanations not required.

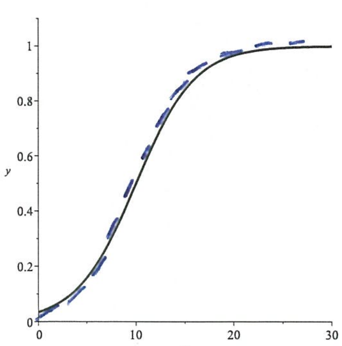
T F If $f(-2) = 8$ and $f(3) = -4$ and f is a continuous function, then the Intermediate Value Theorem guarantees that $f(x)$ will go through $y = 6$ for some value of x in the interval $(-2, 3)$.

6 is between 8 & −4 & f is continuous

T F If the graph of a function $y = f(x)$ is given below, then the derivative $f'(1) = 0$.

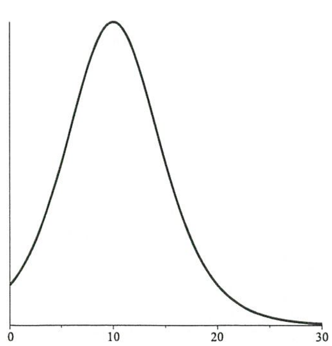


T F Given the graph of $y = f(x)$ below left, its derivative is given by the graph below it.



$y = f(x)$

Tangent line slopes are always positive. They start off getting steeper (larger pos. slope) and then get shallower again (small pos. slopes)



Is this the derivative $f'(x)$?