1. If the polynomial $q(x)$ in the denominator is of higher order, then $\lim_{x\to\pm\infty} \frac{p(x)}{q(x)} = \underline{\quad 0 \quad}$

<u>Ex</u> $\lim_{x\to\infty} \frac{3x^3+x}{x^4-1} = \lim_{x\to\infty} \frac{x^4\left(\frac{3}{x}+\frac{1}{x^3}\right)}{x^4\left(1-\frac{1}{x^4}\right)} = \lim_{x\to\infty} \frac{\frac{3}{x}+\frac{1}{x^3}}{1-\frac{1}{x^4}} = \frac{0+0}{1-0} = \frac{0}{1} = 0$

2. If the polynomial $p(x)$ in the numerator is of higher order, then $\lim_{x\to\pm\infty} \frac{p(x)}{q(x)} = \underline{\quad +\infty \text{ or } -\infty \quad}$

<u>Ex</u> $\lim_{x\to\infty} \frac{2x^3-4x}{-3x^2+2} = \lim_{x\to\infty} \frac{x^2\left(2x-\frac{4}{x}\right)}{x^2\left(-3+\frac{2}{x^2}\right)} = \lim_{x\to\infty} \frac{2x-\frac{4}{x}}{-3+\frac{2}{x^2}} = \frac{\infty-0}{-3+0} = \frac{\infty}{-3} = -\infty$

3. If the polynomials $p(x)$ and $q(x)$ are of the same order, then $\lim_{x\to\pm\infty} \frac{p(x)}{q(x)} = \underline{\quad \text{ratio of leading order coefficients} \quad}$

<u>Ex</u> $\lim_{x\to\infty} \frac{ax^3-4x^2-x}{bx^3+2x-5} = \lim_{x\to\infty} \frac{x^3\left(a-\frac{4}{x}-\frac{1}{x^2}\right)}{x^3\left(b+\frac{2}{x^2}-\frac{5}{x^3}\right)} = \lim_{x\to\infty} \frac{a-\frac{4}{x}-\frac{1}{x^2}}{b+\frac{2}{x^2}-\frac{5}{x^3}} = \frac{a-0-0}{b+0-0+0} = \frac{a}{b}$

(a). $\lim_{x\to\infty} \frac{x-3x^2}{x^3+1}$

(b). $\lim_{x\to\infty} \frac{x^2-x+1}{2x^2+1}$

(c). $\lim_{x\to-\infty} \frac{1+2x+4x^3}{3x-2x^3}$

(d). $\lim_{x\to\infty} \frac{2x^3+5}{3x^2+1}$

(e). $\lim_{x\to\infty} \frac{\sqrt{3x^2-2x}}{x-1}$

(f). $\lim_{x\to 0} \frac{3x^3+2x-5}{x^3+7x+1}$