1. For each pair of functions f and g, find the composite function $F = f \circ g$. [Y

(a).
$$f(x) = x^4 + x^2$$
, $g(x) = 3x - 2x^5$ (b). $f(x) = x^8$, $g(x) = \frac{\sin x}{x - 1}$

(c).
$$f(x) = \sqrt{x}, g(x) = \tan x$$
 (d). $f(x) = \tan x, g(x) = \sqrt{x}$

2. For each function F below, find a pair of functions f and g such that $F = f \circ g$.

(a).
$$F(x) = (6x^2 - 2x + 3)^2 - 4$$
 (b). $F(x) = \left(\frac{5 - 3x}{x + 2}\right)^9$

(c).
$$F(x) = 3\sin x - \sqrt{\sin x}$$
 (d). $F(x) = \tan(\pi x + 1)$

3. A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius (in inches) of the outer circle is given by r(t) = 10t, where t is the time (in seconds) after the pebble strikes the water. The area of a circle is given by the function $A(r) = \pi r^2$.

(a). Find
$$(A \circ r)(t) = A(r(t))$$

(b). Fill in the blank to explain in words what it means $A \circ r$ means: The expression from part (a)

A =_____ gives the _____ of the outer circle as a function of _____.

[You do not need to simplify.]

- 4. Let $f(x) = x^2 + 5x 3$ and $g(x) = 3x^2 + 2x$.
- (a). Find $F(x) = (f \circ g)(x)$. Simplify/expand your answer.

(b). Find F'(x).

(c). Find f'(x) and g'(x).

(d). Find f'(g(x)) [i.e. the composition of $f' \circ g$].

(e). Find $f'(g(x)) \cdot g'(x)$ and simplify your answer.

(f). Compare the result of part (b) with part (e):

TRUE OF FALSE: $F'(x) = f'(g(x)) \cdot g'(x)$

Graph of $f(x) = \sin(x)$:



Graph of $F(x) = \sin(4x)$:



5. Use the graphs above to help answer the following questions.

- (a). How many completes cycles does $f(x) = \sin(x)$ make in the interval $[-\pi, \pi]$?
- (b). How many completes cycles does $f(x) = \sin(4x)$ make in the interval $[-\pi, \pi]$?
- (c). So $F(x) = \sin(4x)$ is changing at a rate that is ______ times as fast as the rate $f(x) = \sin(x)$ changes.
- (d). Since the _____ represents the rate of change, we expect the derivative of $F(x) = \sin(4x)$ to be _____ times as large as the derivative of $f(x) = \sin(x)$.

(e). Sketch the tangent line to $f(x) = \sin(x)$ at x = 0. Estimate the slope of this tangent line:

- (f). Sketch the tangent line to $f(x) = \sin(4x)$ at x = 0. Estimate the slope of this tangent line:
- (g). Do your answers to parts (e) and (f) confirm your guess in part (d)?

6. Suppose the graphs on the next page are given for a car company where

- c(w) = number of cars produced by w workers and
- p(c) = profit in dollars from producing c cars.
- (a). Let $P(w) = (p \circ c)(w) = p(c(w)) = \text{profit from } w \text{ workers.}$
 - (i) If there are w = 200 workers, how many cars c are produced?
 - (ii) If c is the number of cars found in part (i), what is the profit p?
 - (*iii*) Use parts (*i*) and (*ii*) to determine the profit P when you have 200 workers, i.e. find $P(200) = (p \circ c)(200) = p(c(200))$.
 - (iv) Repeat parts (i)-(iii) to find P(0), P(100) and P(300).
 - (v) Use the results of (iii)-(iv) to complete the following table.

w	P(w)
0	
100	
200	
300	

(vi) Use the table to sketch the graph for $P(w) = (p \circ c)(w) = p(c(w))$ on following page.

- (b). If c'(200) = 10, then the slope of the tangent line at w = 200 is ______. So if 200 workers are currently working, approximately how many more cars will be produced by adding one more worker?
- (c). If p'(4000) = 450, then the slope of the tangent line at c = 4000 is ______. So if 4000 cars are currently being produced, approximately how much more profit will be made by producing one more car?
- (d). Based on your answers to parts (b) and (c), fill in the following blanks: If 200 workers are currently working and you add more workers, it will result in _____ more cars per worker and _____ profit per car. So the overall increase to profit is [Fill in the correct numbers below.]

 $\frac{\text{cars}}{\text{worker}} \times \frac{\text{profit}(\$)}{\text{cars}} = \frac{\text{profit}(\$)}{\text{workers}}$

- (e). Based on your answer to part (d), $P'(200) = _$. [i.e. The change in profit P(w) from adding one more worker to the current 200 working.]
- (f). Use your answers from (d) and (e) to write a relationship between P'(200), c'(200), and p'(4000).



