

Basic Limits

1. $\lim_{x \rightarrow a} c = c$

2. $\lim_{x \rightarrow a} x = a$

Limit Laws Suppose $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist and c is a constant, then

3. $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$

4. $\lim_{x \rightarrow a} cf(x) = c \lim_{x \rightarrow a} f(x)$

5. $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$

6. $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}, \text{ if } \lim_{x \rightarrow a} g(x) \neq 0$

7. $\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$ for positive integer n

Even More Special Limits and Laws

8. $\lim_{x \rightarrow a} x^n = a^n$ for positive integer n

9. $\lim_{x \rightarrow a} x^{1/n} = \lim_{x \rightarrow a} \sqrt[n]{x} = \sqrt[n]{a}$ for positive integer n and if n is even, $a \geq 0$

10. $\lim_{x \rightarrow a} [f(x)]^{1/n} = \lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$ for positive integer n . [In the case that n is even, $f(x) \geq 0$]

Ex: Evaluate the following limit, justifying each step with limit laws.

$$\lim_{x \rightarrow 2} \frac{3x^2 + 2x + 2}{\sqrt{2x - 1}} = \frac{\lim_{x \rightarrow 2} 3x^2 + 2x + 2}{\lim_{x \rightarrow 2} \sqrt{2x - 1}} \quad \text{by Law 6}$$

$$= \frac{\lim_{x \rightarrow 2} 3x^2 + \lim_{x \rightarrow 2} 2x + \lim_{x \rightarrow 2} 2}{\sqrt{\lim_{x \rightarrow 2} (2x - 1)}} \quad \text{by Law 3 \& 10}$$

$$= \frac{3 \lim_{x \rightarrow 2} x^2 + 2 \lim_{x \rightarrow 2} x + \lim_{x \rightarrow 2} 2}{\sqrt{2 \lim_{x \rightarrow 2} x - \lim_{x \rightarrow 2} 1}} \quad \text{by Law 3 \& 4}$$

$$= \frac{3(2)^2 + 2(2) + 2}{\sqrt{2(2) - 1}} \quad \text{by Law 8, 2, \& 1}$$