

1. Given $f(\theta) = \cos^2(\theta)$, on $0 \leq \theta \leq 2\pi$, find the intervals of concavity and the inflection points.

Up: $(\frac{\pi}{4}, \frac{3\pi}{4}) \cup (\frac{5\pi}{4}, \frac{7\pi}{4})$

Down: $(0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \frac{5\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$

Inf. Pts.: $(\frac{\pi}{4}, \frac{1}{2}), (\frac{3\pi}{4}, \frac{1}{2}), (\frac{5\pi}{4}, \frac{1}{2}), (\frac{7\pi}{4}, \frac{1}{2})$

2. Evaluate the following limits. [Show all work - no shortcuts].

(a). $\lim_{x \rightarrow \infty} \frac{3 - x^2 + 4x^3}{x^4 + 2x} = 0$

(b). $\lim_{x \rightarrow -\infty} \frac{2x^2 + 3x + 1}{3x^2 - 3x - 4} = \frac{2}{3}$

3. Find $\lim_{x \rightarrow \pm\infty} f(x)$ for the following functions and determine any horizontal and slant asymptotes.

(a). $f(x) = \frac{2x + 1}{\sqrt{4x^2 - x}}$

(b). $g(x) = \frac{2x^3 - 3x^2 + 2}{x^2 - 3x}$

(a). $\lim_{x \rightarrow \infty} f(x) = 1; \lim_{x \rightarrow -\infty} f(x) = -1$

HA: $y = 1$ and $y = -1$

(b) $\lim_{x \rightarrow \infty} g(x) = \infty; \lim_{x \rightarrow -\infty} g(x) = -\infty$

SA: $y = 2x + 3$

4. Given the following function and its derivatives

$$f(x) = \frac{x}{9 - x^2}$$

$$f'(x) = \frac{x^2 + 9}{(9 - x^2)^2}$$

$$f''(x) = \frac{2x(x^2 + 27)}{(9 - x^2)^3}$$

Use the Summary of Curve Sketching to determine the relevant information. Sketch the graph of the function. Label any maximum and minimum points and inflection points.

domain: All real numbers, except $x \neq \pm 3$

slant asymptote: none

coordinates of local max/min(s): none

x-intercept(s): $x = 0$

critical numbers: none

intervals where concave up: $(\infty, -3) \cup (0, 3)$

y-intercept: $y = 0$

intervals where increasing:
 $(\infty, -3) \cup (-3, 3) \cup (3, \infty)$

intervals where concave down:
 $(-3, 0) \cup (3, \infty)$

vertical asymptote(s): $x = 3, x = -3$

intervals where decreasing: none

inflection point(s): $(0, 0)$

Use your calculator to graph the function to check and see if you sketched it correctly.

5. If a box with a square base and open top is to hold 4 ft^3 , find the dimensions of the box that will require the least amount of material.

$$2' \times 2' \times 1'$$

6. Find the maximum possible volume of a right circular cylinder if its total surface area (including top and bottom) is 150π .

$$V = 250\pi \quad (r = 5, h = 10)$$

7. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions of the Norman window with the largest possible area if the total perimeter is 16 ft.

$$\text{Rect. portion: } \frac{16}{\pi+4} \times \frac{32}{\pi+4} \text{ ft.}$$

8. Given $f(x) = 4x^3 - 12x^2 + 12x - 3$

(a). Explicitly write out Newton's formula for finding the root of this function.

$$x_{n+1} = x_n - \frac{4x_n^3 - 12x_n^2 + 12x_n - 3}{12x_n^2 - 24x_n + 12}$$

(b). Starting with $x_0 = 0.5$, demonstrate Newton's method by marking $x_0, x_1, x_2, \dots$ and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess? Yes

(c). Starting with $x_0 = 1.0$, demonstrate Newton's method by marking $x_0, x_1, x_2, \dots$ and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess? Why or why not?

No, it won't work since the tangent line is horizontal when $x_0 = 1.0$.

9. Find the antiderivatives for the following functions.

(a). $h(x) = 3x^3 - 7x^2$

$$H(x) = \frac{3}{4}x^4 - \frac{7}{3}x^3 + C$$

(b). $f(x) = \sqrt{x} - \sqrt{3}x^2$

$$F(x) = \frac{2}{3}x^{3/2} - \frac{\sqrt{3}}{3}x^3 + C$$

10. Given that $g'(\theta) = -\sec^2 \theta$ and $g\left(\frac{\pi}{3}\right) = 0$, find $g(\theta)$.

$$g(\theta) = -\tan \theta + \sqrt{3}$$

11. Given the function $f(x) = \frac{3}{x}$, estimate the area under the curve $f(x)$ on the interval $[1, 6]$ using 5 subintervals and using the right endpoint of each subinterval. [i.e. find R_5]. You must actually find the value.

$$R_5 = \Delta x \cdot [f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] = 1 \cdot [f(2) + f(3) + f(4) + f(5) + f(6)] = 1 \cdot [3/2 + 3/3 + 3/4 + 3/5 + 3/6] = \frac{87}{20}$$

12. Using the definition of the definite integral $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} R_n$, set-up, but do not evaluate,

the summation/limit using right endpoints for the integral $\int_0^1 x^3 + 1 dx$.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left[\left(\frac{i}{n} \right)^3 + 1 \right]$$

13. Evaluate the limit $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left(\frac{i}{n} \right)^2 = \frac{1}{3}$

14. Section 4.3: #3

15. Evaluate the following integrals [Use integration techniques, not the limit definition.]:

$$(a). \int_1^2 t + 2 dt = \frac{7}{2}$$

$$(d). \int_1^{x^2} t + 2 dt = \frac{1}{2}x^4 + 2x^2 - \frac{5}{2}$$

$$\int_0^4 \frac{x(2+x)}{\sqrt{x}} dx = \frac{352}{15}$$

16. Use the Fundamental Theorem of Calculus (Part B/1) to find $F'(x)$

$$(a). F(x) = \int_0^x t \cos t dt$$

$$F'(x) = \frac{d}{dx} \left[\int_0^x t \cos t dt \right] = x \cos x$$

$$(b). F(x) = \int_{-2}^{x^2} \sqrt{t+8} dt$$

$$F'(x) = \frac{d}{dx} \left[\int_{-2}^{x^2} \sqrt{t+8} dt \right] = \sqrt{x^2+8} \cdot 2x = 2x\sqrt{x^2+8}$$

17. Evaluate the following integrals.

$$(a). \int 3x^5 - 4x^3 + 6x + 2 dx = \frac{1}{2}x^6 - x^4 + 3x^2 + 2x + C$$

$$(d). \int_0^{\pi/6} \sec x \tan x dx = \frac{2}{\sqrt{3}} - 1$$

$$(b). \int x(3x^2 - 2x)^2 dx = \frac{3}{2}x^6 - \frac{12}{5}x^5 - x^4 + C$$

$$(e). \int_1^3 \frac{x^2+1}{x^2} dx = \frac{8}{3}$$

$$(c). \int \left(1 + \frac{1}{t}\right) \left(\frac{1}{t^2}\right) dt = -\frac{1}{t} - \frac{1}{2t^2} + C$$

$$(f). \int y^2 \sqrt{y} dy = \frac{2}{7}y^{7/2} + C$$

18. A particle moves with a velocity of $v(t) = -t^2 + 4t$ on the interval $0 \leq t \leq 6$.

(a). Find the displacement

$$0$$

(b). Find the total distance traveled

$$\frac{32}{3} + \left| -\frac{32}{3} \right| = \frac{64}{3}$$

19. Let $r(t)$ be the rate at which the world's oil is consumed, where t is measured in years starting at $t = 0$ on January 1, 2000 and $r(t)$ is measured in barrels per year. What does $\int_0^8 r(t) dt$ represent and what are its units?
The integral represents the change in the amount of oil consumed from 2000 to 2008. The units are barrels.