

1. Given $f(\theta) = \cos^2(\theta)$, on $0 \leq \theta \leq 2\pi$, find the intervals of concavity and the inflection points.

2. Evaluate the following limits. [Show all work - no shortcuts].

(a). $\lim_{x \rightarrow \infty} \frac{3 - x^2 + 4x^3}{x^4 + 2x}$

(b). $\lim_{x \rightarrow -\infty} \frac{2x^2 + 3x + 1}{3x^2 - 3x - 4}$

3. Find $\lim_{x \rightarrow \pm\infty} f(x)$ for the following functions and determine any horizontal and slant asymptotes.

(a). $f(x) = \frac{2x + 1}{\sqrt{4x^2 - x}}$

(b). $g(x) = \frac{2x^3 - 3x^2 + 2}{x^2 - 3x}$

4. Given the following function and its derivatives

$$f(x) = \frac{x}{9 - x^2} \quad f'(x) = \frac{x^2 + 9}{(9 - x^2)^2} \quad f''(x) = \frac{2x(x^2 + 27)}{(9 - x^2)^3}$$

Use the Summary of Curve Sketching to determine the relevant information. Sketch the graph of the function. Label any maximum and minimum points and inflection points.

domain:	slant asymptote:	coordinates of local max/min(s):
x-intercept(s):	critical numbers:	intervals where concave up:
y-intercept:	intervals where increasing:	intervals where concave down:
vertical asymptote(s):		
horizontal asymptote(s):	intervals where decreasing:	inflection point(s):

5. If a box with a square base and open top is to hold 4 ft³, find the dimensions of the box that will require the least amount of material.

6. Find the maximum possible volume of a right circular cylinder if its total surface area (including top and bottom) is 150π .

7. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions of the Norman window with the largest possible area if the total perimeter is 16 ft.

8. Given $f(x) = 4x^3 - 12x^2 + 12x - 3$

- (a). Explicitly write out Newton's formula for finding the root of this function.
- (b). Starting with $x_0 = 0.5$, demonstrate Newton's method by marking x_0, x_1, x_2, \dots and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess?
- (c). Starting with $x_0 = 1.0$, demonstrate Newton's method by marking x_0, x_1, x_2, \dots and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess? Why or why not?

9. Find the antiderivatives for the following functions.

(a). $h(x) = 3x^3 - 7x^2$

(b). $f(x) = \sqrt{x} - \sqrt{3}x^2$

10. Given that $g'(\theta) = -\sec^2 \theta$ and $g\left(\frac{\pi}{3}\right) = 0$, find $g(\theta)$.

11. Given the function $f(x) = \frac{3}{x}$, estimate the area under the curve $f(x)$ on the interval $[1, 6]$ using 5 subintervals and using the right endpoint of each subinterval. [i.e. find R_5]. You must actually find the value.

12. Using the definition of the definite integral $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} R_n$, set-up, but do not evaluate, the summation/limit using right endpoints for the integral $\int_0^1 x^3 + 1 dx$.

13. Evaluate the limit $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left(\frac{i}{n}\right)^2$

14. Section 4.3: #3

15. Evaluate the following integrals [Use integration techniques, not the limit definition.]:

(a). $\int_1^2 t + 2 dt$

(d). $\int_1^{x^2} t + 2 dt$

$\int_0^4 \frac{x(2+x)}{\sqrt{x}} dx$

16. Use the Fundamental Theorem of Calculus (Part B/1) to find $F'(x)$

(a). $F(x) = \int_0^x t \cos t dt$

(b). $F(x) = \int_{-2}^{x^2} \sqrt{t+8} dt$

17. Evaluate the following integrals.

(a). $\int 3x^5 - 4x^3 + 6x + 2 dx$

(d). $\int_0^{\pi/6} \sec x \tan x dx$

(b). $\int x(3x^2 - 2x)^2 dx$

(e). $\int_1^3 \frac{x^2 + 1}{x^2} dx$

(c). $\int \left(1 + \frac{1}{t}\right) \left(\frac{1}{t^2}\right) dt$

(f). $\int y^2 \sqrt{y} dy$

18. A particle moves with a velocity of $v(t) = -t^2 + 4t$ on the interval $0 \leq t \leq 6$.

(a). Find the displacement

(b). Find the total distance traveled

19. Let $r(t)$ be the rate at which the world's oil is consumed, where t is measured in years starting at $t = 0$ on January 1, 2000 and $r(t)$ is measured in barrels per year. What does $\int_0^8 r(t) dt$ represent and what are its units?