

1. Differentiate the following using Differentiation Rules

(a). $s(t) = (3t^3 - t^2 + 7)^{23}$

$$s'(t) = 23(3t^3 - t^2 + 7)^{22}(9t^2 - 2t)$$

(b). $y = \frac{x(2x^4 + 4)^8}{\tan 2x}$ [Do not simplify!]

$$\frac{dy}{dx} = \frac{\tan 2x \cdot [x \cdot 8(2x^4 + 4)^7 \cdot 8x^3 + (2x^4 + 4)^8 \cdot 1] - x(2x^4 + 4)^8 (\sec^2(2x) \cdot 2)}{\tan^2 2x}$$

(c). $f(\theta) = \theta \sin(\theta^2 + 1)$

$$f'(\theta) = 2\theta^2 \cos(\theta^2 + 1) + \sin(\theta^2 + 1)$$

2. Find the equation of the tangent line to the curve $y = \sqrt[3]{2x^2 - 5}$ at $x = 4$.

$$y - 3 = \frac{16}{27}(x - 4)$$

3. Given $f(x) = g(3x^2)$, find f' in terms of g' .

$$f'(x) = g'(3x^2) \cdot 6x$$

4. If a stone is thrown vertically upward on the moon with a velocity of 8 m/s, its height after t seconds is given by $y = 8t - 0.83t^2$

(a). What is the velocity after 2 s?

$$4.68 \text{ m/s}$$

(b). What is the velocity at impact?

$$-8 \text{ m/s}$$

5. A tank holds 1000 gallons of water, which drains from the bottom of the tank in 50 minutes. Torricelli's Law gives the volume V of water remaining in the tank after t minutes as $V = 1000 \left(1 - \frac{1}{50}t\right)^2$ for $0 \leq t \leq 50$. Find the rate at which the water is draining from the tank after 10 minutes. Include units in your answer. -32 gallons/min.

6. The cost function for a certain commodity is $C(x) = 60 + 0.12x - 0.0004x^2 + .000002x^3$.

(a). Find the marginal cost function.

$$C'(x) = 0.12 - 0.0008x + 0.000006x^2$$

(b). Find and interpret $C'(50)$.

$$C'(50) = \$0.095. \text{ The rate the cost is changing when the 50th item is produced is approximately } \$0.095 \text{ per item.}$$

(c). Compare $C'(50)$ with the cost of producing the 51st item.

$$C(51) - C(50) = \$0.094902$$

7. Any Section 2.7 applications.

8. Given the curve drawn below and defined by

$$x^2 + y^2 = 3 + xy$$

(a). Find $\frac{dy}{dx}$

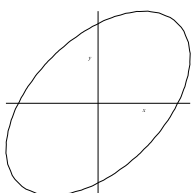
$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

(b). On the graph below, sketch any tangents lines to the curve where the slope is 0.

(c). Use part (a) to find these points on the curve where the slope is 0. Must show work for credit. $(1, 2)$ & $(-1, -2)$.

(d). Find $\frac{d^2y}{dx^2}$ in terms of x and y .

$$\frac{d^2y}{dx^2} = \frac{(2y - x) \left(\frac{y - 2x}{2y - x} - 2 \right) - (y - 2x) \left(2 \frac{y - 2x}{2y - x} - 1 \right)}{(2y - x)^2}$$



9. Given $f(x) = \sqrt{x} = x^{1/2}$

(a). Find the linearization $L(x)$ at $a = 25$

$$L(x) = 5 + \frac{1}{10}(x - 25)$$

(b). Use this linearization $L(x)$ to approximate $\sqrt{24.7}$ [Simplify your answer.]

4.97

(c). Find the differential dy for x going from 25 to 25.5.

0.05

10. A ladder 8 feet long is leaning against the wall of a house. On the ground, the base of the ladder is being pulled away from the wall at a rate of $\frac{3}{2}$ ft/sec. How fast is the angle between the top of the ladder and the wall changing when the this angle is $\frac{\pi}{3}$.

$\frac{3}{8}$ rad/s

11. A particle moves along the curve $xy^2 = 12$. As it reaches the point $(3, 2)$, the y -coordinate is decreasing at a rate of 2 cm/s. How fast is the x -coordinate of the particle position changing at that instant?

6 cm/s

12. Find the critical numbers for $g(t) = 4t^3 - 3t^2 + 1$

$$t = 0, \frac{1}{2}$$

13. Given $f(x) = 2\sqrt{x} - x$, find the absolute maximum and minimum values of $f(x)$ on the interval $[0, 2]$.

Absolute Max = 1 at $x = 1$; Absolute Min = 0 at $x = 0$.

14. Given $f(x) = \frac{(x-1)^3}{x^2}$

(a). Find the intervals of increase or decrease.

Increasing: $(-\infty, -2) \cup (0, 1) \cup (1, \infty)$

Decreasing: $(-2, 0)$

(b). Find the local maximum and minimum values.

No Min ; Max = $-\frac{27}{4}$ at $x = -2$.

(c). Find the intervals of concavity and the inflection points.

Up: $(1, \infty)$ Down: $(-\infty, 0) \cup (0, 1)$ Inf. Pts.: $(1, 0)$

15. Given $f(\theta) = \cos^2(\theta)$, on $0 \leq \theta \leq 2\pi$,

(a). Find the intervals of increase or decrease.

Increasing: $(\frac{\pi}{2}, \pi) \cup (\frac{3\pi}{2}, 2\pi)$

Decreasing: $(0, \frac{\pi}{2}) \cup (\pi, \frac{3\pi}{2})$

(b). Find the local maximum and minimum values.

Min = 0 at $x = \frac{\pi}{2}, \frac{3\pi}{2}$ and Max = 1 at $x = \pi$.

(c). Find the intervals of concavity and the inflection points.

Up: $(\frac{\pi}{4}, \frac{3\pi}{4}) \cup (\frac{5\pi}{4}, \frac{7\pi}{4})$

Down: $(0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \frac{5\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$

Inf. Pts.: $(\frac{\pi}{4}, \frac{1}{2}), (\frac{3\pi}{4}, \frac{1}{2}), (\frac{5\pi}{4}, \frac{1}{2}), (\frac{7\pi}{4}, \frac{1}{2})$

16. Section 3.3 #27

17. Apply the Mean Value Theorem to the function $f(x) = \sqrt{x-2}$ on the interval $[2, 6]$ and find all values of c that satisfy the MVT.

$c = 3$