By definition, $$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_{a}^{\infty} f(x)\,dx$$

where a is typically chosen to be 0 and each semi-infinite integral is evaluated (by definition) as a separate limit:

$$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{R_1\to-\infty}\int_{R_1}^{0} f(x)\,dx + \lim_{R_2\to\infty}\int_{0}^{R_1} f(x)\,dx$$

If both of these limits exist (as finite real numbers), then $\int_{-\infty}^{\infty} f(x)\,dx$ converges to the sum of those values.

If either or both of these limits diverge, then the improper integral $\int_{-\infty}^{\infty} f(x)\,dx$ diverges.

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1. Given $\int_{-\infty}^{\infty} 2x^3\,dx$

(a). Evaluate $\int_{-\infty}^{\infty} 2x^3\,dx$ correctly and determine whether it converges or diverges.
If it converges, state the value it converges to.

(b). Evaluate $\lim_{R\to\infty}\int_{-R}^{R} 2x^3\,dx$. [Evaluate the integral and **simplify**. Then take the limit.]

(c). Does $\int_{-\infty}^{\infty} 2x^3\,dx = \lim_{R\to\infty}\int_{-R}^{R} 2x^3\,dx$?

Helpful Hints: $e^{2x} = (e^x)^2$ $\quad \int \frac{1}{u^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)$ Be extra careful when evaluating the limits.

2. Given $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{e^{2x}+9}\,dx$

(a). Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{e^{2x}+9}\,dx$ correctly and determine whether it converges or diverges.
If it converges, state the value it converges to.

(b). Evaluate $\displaystyle\lim_{R\to\infty}\int_{-R}^{R} \frac{e^x}{e^{2x}+9}\,dx.$ [Evaluate the integral and **simplify**. Then take the limit.]

(c). Does $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{e^{2x}+9}\,dx = \lim_{R\to\infty}\int_{-R}^{R} \frac{e^x}{e^{2x}+9}\,dx?$