[Problems #1-4 do on separate paper. Problems #5-6 fill in the blanks on this worksheet and have completed for next time!]

Recall, $\displaystyle\sum_{n=0}^{\infty} az^n = \frac{a}{1-z}$, for $|z| < 1$ and $\displaystyle\sum_{n=m}^{\infty} az^n = \frac{az^m}{1-z}$, for $|z| < 1$.

1. Determine whether the following series converge or diverge. If it converges, find the sum.

(a). $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2}{1+i}\right)^n$ **(b).** $\displaystyle\sum_{n=0}^{\infty} \frac{4^n}{(2-i)^{2n}}$ **(c).** $\displaystyle\sum_{n=3}^{\infty} \left(\frac{i}{2}\right)^n$

2. Determine for which values of z the following series converges and find (and simplify) the sum.

$$\sum_{n=0}^{\infty} \frac{3(z-i)^n}{2^n}$$

3. Rewrite the following functions as a geometric series and state where they converge.

(a). $\dfrac{-3}{5+z}$ **(b).** $\dfrac{4}{2z-1}$ **(c).** $\dfrac{1}{s-z}$

4. In class we proved that the geometric series $\displaystyle\sum_{n=0}^{\infty} az^n = \frac{a}{1-z}$, for $|z| < 1$.

Use a similar process to prove the more general result for the series starting at any integer $m \geq 0$. That is, show

$$\sum_{n=m}^{\infty} az^n = \frac{az^m}{1-z}, \quad \text{for } |z| < 1$$

5. Recall from class that the function $\dfrac{1}{1-z} = \displaystyle\sum_{n=0}^{\infty} z^n$. Fill in the blanks below to rewrite the function as the sum of the $N-1$ partial sum S_{N-1} and a remainder term R_N.

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n = \quad 1 + z + z^2 + z^3 + \cdots$$

$$= \underbrace{1 + z + z^2 + z^3 + \cdots + z^{N-1}}_{S_{N-1}} + \underbrace{z^N + z^{N+1} + z^{N+3} + \cdots}_{R_N}$$

$$= \sum_{n=0} z^n + \sum_{n=}^{\infty} z^n$$ ⟵ Fill in the missing bounds.

$$= \sum_{n=0} z^n + \frac{\quad}{1-z}$$ ⟵ Fill in the numerator using the general result proved in #4.

6. From problem 3(c) you should have gotten $\dfrac{1}{s-z} = \dfrac{1}{s}\sum_{n=0}^{\infty}\left(\dfrac{z}{s}\right)^n$. Fill in the blanks below to rewrite the function as the sum of the $N-1$ partial sum S_{N-1} and a remainder term R_N.

$$\frac{1}{s-z} = \frac{1}{s}\sum_{n=0}^{\infty}\left(\frac{z}{s}\right)^n = \frac{1}{s}\left[1+\frac{z}{s}+\left(\frac{z}{s}\right)^2+\left(\frac{z}{s}\right)^3+\cdots\right]$$

$$= \frac{1}{s}\left[\underbrace{1+\frac{z}{s}+\left(\frac{z}{s}\right)^2+\left(\frac{z}{s}\right)^3+\cdots+\left(\frac{z}{s}\right)^{N-1}}_{S_{N-1}}+\underbrace{\left(\frac{z}{s}\right)^{N}+\left(\frac{z}{s}\right)^{N+1}+\left(\frac{z}{s}\right)^{N+3}+\cdots}_{R_N}\right]$$

$$= \frac{1}{s}\left[\sum_{n=0}\left(\frac{z}{s}\right)^n + \sum_{n=}^{\infty}\left(\frac{z}{s}\right)^n\right] \qquad \longleftarrow \text{Fill in the missing bounds.}$$

$$= \frac{1}{s}\left[\sum_{n=0}\left(\frac{z}{s}\right)^n\right] + \frac{1}{s}\left[\sum_{n=}^{\infty}\left(\frac{z}{s}\right)^n\right] \qquad \longleftarrow \text{Fill in the missing bounds.}$$

$$= \frac{1}{s}\left[\sum_{n=0}^{N-1}\left(\frac{z}{s}\right)^n\right] + \frac{1}{s}\left[\frac{\quad}{1-\left(\frac{z}{s}\right)}\right] \qquad \longleftarrow \text{Fill in the numerator using the result proved in \#4.}$$

$$= \frac{1}{s}\left[\sum_{n=0}^{N-1}\left(\frac{z}{s}\right)^n\right] + \frac{1}{s}\left[\frac{z^N}{s^N\left(1-\frac{z}{s}\right)}\right] \qquad \longleftarrow \text{Check your work so far.}$$

$$= \frac{1}{s}\left[\sum_{n=0}^{N-1}\frac{\quad}{s^n}\right] + \frac{z^N}{s^N\cdot s\left(1-\frac{z}{s}\right)} \qquad \longleftarrow \text{Fill in the missing numerator.}$$

$$\left[\sum_{n=0}^{N-1}\frac{z^n}{\qquad}\right] + \frac{z^N}{s^N\left(s-\quad\right)} \qquad \longleftarrow \text{Distribute } s \text{ and fill in denominator terms}$$

i.e.

$$\boxed{\frac{1}{s-z} = \qquad\qquad\qquad\qquad (*)}$$

Additonal Book Homeowork:
8th Ed. Section 5.55-56, p. 188 (9th Ed. Section 5.60-61, p. 185): #4, 6-8[You may use corresponding theorems for real numbers.]