1. Given $f(z) = \dfrac{1}{z^2}$

(a). What is the natural domain of f?

(b). Is this domain simply connected? Why or why not?

(c). Is f analytic on its natural domain?

(d). If C is the unit circle $e^{i\theta}$, $0 \le \theta \le 2\pi$, is f analytic on the region bounded by C?

Can you use the Cauchy-Goursat Theorem to evaluate $\displaystyle\int_C \frac{1}{z^2}\,dz$?

(e). Evaluate $\displaystyle\int_C \frac{1}{z^2}\,dz$ [By parameterization.]

$\displaystyle\int_C \frac{1}{z^2}\,dz =$ where C is the unit circle.

(f). If C is the square centered at the origin and beginning and ending at the point (1,0), is f analytic on the region bounded by C? [Note: Same initial and terminal points as unit circle.]

Can you use the Cauchy-Goursat Theorem to evaluate $\int_C \frac{1}{z^2}\,dz$?

(g). Evaluate $\int_C \frac{1}{z^2}\,dz$ [By parameterization – see picture.]

$\int_{C_2} \frac{1}{z^2}\,dz =$

$\int_{C_3} \frac{1}{z^2}\,dz =$

$\int_{C_4} \frac{1}{z^2}\,dz =$

$\int_{C_5+C_1} \frac{1}{z^2}\,dz =$

$\int_C \frac{1}{z^2}\,dz =$	where C is the given square.

2. Given $f(z) = \frac{1}{z}$

(a). What is the natural domain of f?

(b). Is this domain simply connected? Why or why not?

(c). Is f analytic on its natural domain?

(d). If C is the unit circle $e^{i\theta}$, $0 \le \theta \le 2\pi$, is f analytic on the region bounded by C?

Can you use the Cauchy-Goursat Theorem to evaluate $\int_C \frac{1}{z}\,dz$?

(e). Evaluate $\int_C \frac{1}{z}\,dz$ [By parameterization.]

$\int_C \frac{1}{z}\,dz =$ where C is the unit circle.

(f). If C is the square centered at the origin and beginning and ending at the point (1,0), is f analytic on the region bounded by C? [Note: Same initial and terminal points as unit circle.]

Can you use the Cauchy-Goursat Theorem to evaluate $\int_C \frac{1}{z}\, dz$?

(g). Evaluate $\int_C \frac{1}{z^2}\, dz$ [By parameterization – see picture.]

[If short on time, make an educated guess for what you think the final answer will be without actually doing the work.]

$\int_{C_2} \frac{1}{z}\, dz =$

$\int_{C_3} \frac{1}{z}\, dz =$

$\int_{C_4} \frac{1}{z}\, dz =$

$\int_{C_5+C_1} \frac{1}{z}\, dz =$

$\int_C \frac{1}{z}\, dz =$ where C is the given square.

In fact, $\int_C \frac{1}{z^2}\, dz = 0$ and $\int_C \frac{1}{z}\, dz = 2\pi i$ for ***all*** simple closed curves C around the origin.

We will explore the following two questions arising from these examples:

(a). Why did $\int_C \frac{1}{z^2}\, dz = 0$, whereas $\int_C \frac{1}{z}\, dz \neq 0$ for all closed contours?

(b). Why did $\int_C \frac{1}{z^2}\, dz = 0$ for both the circle and the square? Why did $\int_C \frac{1}{z}\, dz = 2\pi i$ for both the circle and the square?