Definite Integral of a real function f over the interval $[a, b]$. [Calc I]

Derivation: Find the area under the curve $f(x) \geq 0$ on the interval $[a, b]$

Divide $[a, b]$ into n subintervals of length $\Delta x =$

Approximate the area with rectangles for each subinterval: $A_k =$

Add up all the rectangles to approximate the total area: $A \approx \sum_{k=1}^{n} f(x_k^*)\Delta x$.

Take the limit and define the definite integral: $\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$

Find the **Arc Length** of a curve defined parametrically by $C : x = x(t),\ y = y(t)$ on the interval $\alpha \leq t \leq \beta$.
[Calc II/III]

Divide the parameter interval $[\alpha, \beta]$ into n subintervals.

So a point on the curve C is given by $P_k =$

Approximate the length by straight line segments in each subinterval:

$$L_k = \sqrt{(x_k - x_{k-1})^2 + (y_k - y_{k-1})^2}$$

$$= \sqrt{(x(t_k) - x(t_{k-1}))^2 + (y(t_k) - y(t_{k-1}))^2}$$

Using the MVT for derivatives: $x'(c_k) =$

$$=$$

$$= \sqrt{[x'(c_k)]^2 + [y'(c_k)]^2}\Delta t_k$$

Add up all the line segments to approximate the total arc length: $L \approx \sum_{k=1}^{n} \sqrt{[x'(c_k)]^2 + [y'(c_k)]^2}\Delta t_k$

Take the limit and define the integral: $L =$ $\quad = \lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{[x'(t_k^*)]^2 + [y'(t_k^*)]^2}\Delta t_k$

Which can also be written alternately as $\quad L = \int_\alpha^\beta ds \quad$ where $ds =$

Line Integrals in the Plane: Integrate f along a parametrically given curve $C : x = x(t), y = y(t)$ for $\alpha \leq t \leq \beta$
[Calc III]

with respect to x with respect to y with respect to arc length s

$$\int_C f(x, y)\ dx \qquad \int_C f(x, y)\ dy \qquad \int_C f(x, y)\ ds$$

Then the limit definitions of the line integrals are:

Line Integral along C with respect to x: $$\int_C f(x, y)\ dx = \lim_{||P|| \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*) \Delta x_k$$

Line Integral along C with respect to y: $$\int_C f(x, y)\ dy = \lim_{||P|| \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*) \Delta y_k$$

Line Integral along C with respect to arc length s: $$\int_C f(x, y)\ ds = \lim_{||P|| \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*) \Delta s_k$$

But we don't need the limit definitions to evaluate the integrals. Instead, convert the line integral to a definite integral with respect to the parameter t on the interval $[\alpha, \beta]$: [C traversed once.]

$x = x(t)$ $y = y(t)$ s is arc length

differentials:

Make the substitutions:

$$\int_C f(x,y)\ dx =$$

$$\int_C f(x,y)\ dy =$$

$$\int_C f(x,y)\ ds =$$