DEF The exponential function for complex numbers is defined by [y in radians]

$$\boxed{\exp(z) = e^z = e^{x+iy} = e^{\,x}\, e^{\,iy}}$$

Some Things Stay the Same

Basic Properties:

$e^{z_1}e^{z_2} = e^{z_1+z_2}$ $\dfrac{e^{z_1}}{e^{z_2}} = e^{z_1-z_2}$ $(e^z)^n = e^{nz}$ $e^0 = 1$ [but...[1]]

The Derivative:

$\dfrac{d}{dz}[e^z] = e^z$

New for e^z in the Complex Plane

- $|e^z| = |e^x e^{iy}|$ by definition of e^z.

  $= |e^x| \cdot$ ___$|e^{iy}|$___ by properties of modulus.

  $= |e^x| \cdot$ ___1___ since e^{iy} lies ___on the unit circle___ .

  $= e^x$ since e^x is a ___positive___ real number.

  $$\boxed{|e^z| = e^x}$$

- Since $\arg z$ is the set of *all* angles θ that represent the same number and $e^z = e^x e^{iy} \Longrightarrow$

  $$\boxed{\begin{aligned} \arg(z) &= y + 2n\pi \\ & n = 0, \pm 1, \pm 2, \ldots \end{aligned}}$$

- $e^{z+2\pi i} = e^z \cdot$ ___$e^{2\pi i}$___ $= e^z \cdot 1 =$ ___e^z___ i.e. $f(z + 2\pi i) = f(z)$ for all z.

  The ***complex*** exponential function $f(z) = e^z$ is ___periodic___ with period ___$2\pi i$___

- Ex: Evaluate $4e^{i\pi} =$ ___-4___

  The ***complex*** exponential function $f(z) = e^z$ may return ___negative___ real numbers.

[1] [The "but..." above refers to the fact that $e^z = 1$ not only at $z = 0$, but also at $z =$ ___$2n\pi i$___.]

Ex: Solve $e^x = 3$ for $x \in \mathbb{R}$.

Is the solution unique? Yes

Ex: Solve $e^z = 3$ for $z \in \mathbb{C}$. Soln: Let $z = x + iy$. Then fill in the blanks below to find x and y.

$$e^z = 3$$

$$e^{x+iy} = 3e^{i(0+2n\pi)} \qquad n = 0, \pm 1, \pm 2, \ldots$$

$$e^x e^{iy} = 3e^{i(0+2n\pi)} \qquad n = 0, \pm 1, \pm 2, \ldots$$

Equating the magnitudes $\Longrightarrow$: $e^x =$ __3__ Equating the argument $\Longrightarrow$: $y =$ __$0 + 2n\pi$__

Solving for x and y $\Longrightarrow$: $x =$ __$\ln 3$__ and $y =$ __$2n\pi$__.

So $z = x + iy =$ __$\ln 3$__ $+ i$ __$(2n\pi)$__, $n = 0, \pm 1, \pm 2, \ldots$

Is the solution unique? No

The above two examples illustrate the following point:

There are __infinitely__ many solutions to the ***complex*** equation $e^z = w$.

Ex: Solve $e^z = 3 - \sqrt{3}i$. [Use the method of the 2nd example above and write your answer in the form $x + iy$.]

Homework 8th Edition Section 3.29, p. 93: #1, 3, 6, 8, 10 9th Edition Section 3.30, p. 89: #1, 3, 6, 8, 10