1. Express the following in the form $a + ib$. $\tanh^{-1}(1+2i)$

2. Solve $\sin z = \sqrt{2}$ using inverse trigonometric functions.

3. Evaluate the following integrals

(a). $\displaystyle\int_1^2 \frac{2i}{t} - t\; dt$

(b). $\displaystyle\int_C G(x,y)\; dx$, and $\displaystyle\int_C G(x,y)\; dy$, for $G(x,y) = 3x^2 + 6y^2$ and $C : x = t, y = t^2 + 1$ for $-1 \le t \le 0$

(c). $\displaystyle\int_C x^6 y\; ds$ where $C : x = t, y = \frac{1}{t}\;\; 1 \le t \le 2$

(d). $\displaystyle\int_C (x^2 + iy^3)\; dz$ where C is the straight line from $z = 1$ to $z = i$.

(e). $\displaystyle\oint_C ze^z\; dz$ where C is the square with vertices $z = 0, z = 1, z = 1 + i, z = i$.

4. By finding an antiderivative, evaluate the following integrals, where the path is an arbitrary contour between the indicated limits of integration.

(a). $\displaystyle\int_0^{3+i} z^2\; dz$ **(b).** $\displaystyle\int_{1-i}^{1+2i} ze^{z^2}\; dz$ **(c).** $\displaystyle\int_{\pi}^{\pi+2i} \sin\frac{z}{2}\; dz$

5. Given the following functions, determine for which ones The Cauchy-Goursat Theorem applies so that $\displaystyle\oint_C f(z)\; dz = 0$. For those which it does not apply, evaluate the integral $\displaystyle\oint_C f(z)\; dz$.

(a). $f(z) = \dfrac{z}{2z+3}$; C: the unit circle $|z| = 1$

(b). $f(z) = \dfrac{z}{2z+3}$; C: the circle $|z| = 2$

(c). $f(z) = \tan z$; C: the unit circle $|z| = 1$

6. Given $\displaystyle\oint_C \frac{5}{z-1-i}\; dz$ and $C : x^4 + y^4 = 16$.

(a). Explain why $\displaystyle\oint_C \frac{5}{z-1-i}\; dz \ne 0$

(b). Over which of the following contours will the integral give you the same answer.
(i). $|z| = 2$ **(ii).** $|z| = 1/2$ **(iii).** $|z - 1 - i| = 1$
(iv). square with vertices at $z = 0, 2, 2 + 2i, 2i$

7. Use the Cauchy Integral formula, when appropriate, to evaluate

(a). $\displaystyle\oint_C \frac{z^2 - 3z + 4i}{z + 2i}\, dz;\ \ C: |z| = 3$

(b). $\displaystyle\oint_C \frac{z^2}{z^2 + 4}\, dz;\ \ C:$ (i) $|z - i| = 2$ (ii) $|z + 2i| = 1$

(c). $\displaystyle\oint_C \frac{2z + 5}{z^2 - 2z}\, dz;\ \ C:$ (i) $|z| = 1/2$ (ii) $|z + 1| = 2$ (iii) $|z - 3| = 2$ (iv) $|z + 2i| = 1$

(d). $\displaystyle\oint_C \frac{2z^3 + 3z}{(z + 2)^3}\, dz;\ \ C: |z - i| = 4$

8. Let C_R denote the quarter circle in quadrant I of $|z| = R (R > \sqrt{5})$, taken in the counterclockwise direction.

(a). Find an upper bound on $\displaystyle\left|\int_{C_R} \frac{3z^2 - 2}{z^2 + 5z^4}\, dz\right|$.

(b). Find the limit of the upper bound found in part (a) as $R \to \infty$.

9. Determine whether the following series converge or diverge. If it converges, find the sum.

(a). $\displaystyle\sum_{n=0}^{\infty} \left(\frac{\pi i}{3}\right)^n$

(b). $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}}{(1 - 2i)^{n-2}}$

10. Determine for which values of z the following series converges and find (and simplify) the sum.

$$\sum_{n=0}^{\infty} \frac{(2z + 6i)^n}{3^n}$$

11. Rewrite the following function as a geometric series and state where it converges. $\displaystyle f(z) = \frac{3}{3z - 4}$

12. Derive the Taylor Series for $\sinh(z)$ by

(a). Using the definition of Taylor Series.

(b). Using the definition $\displaystyle\sinh z = \frac{e^z - e^z}{2}$ and the known Maclaurin Series for e^z.

(c). Using the identity $\sinh z = -i \sin(iz)$ and the known Maclaurin Series for $\sin z$.

13. Use a known Maclaurin Series to find a series expansion for $\displaystyle f(z) = \frac{3z^2}{4z^4 + 5}$ and state the values of z where it converges. Is the resulting series a Maclaurin Series?