Name: ______________________________ **Exam 2**

Math 434, Complex Variables – Crawford **26 April 2015**

- Books and notes (in any form) are not allowed.
- You may use calculators and the provided formula sheet.
- ***Show all your work***. Partial credit may be given for written work.
- Unless otherwise stated,
  - ***Brief*** explanations should only require 1-2 sentences.
  - Simplify/Evaluate trigonometric, exponential, logarithmic, and hyperbolic functions for standard values.
  - Do ***not*** not simplify answers to the form $a+bi$ (unless needed for the answer).
  - Assume that closed contours are positively oriented.
  - Do ***not*** not shift indices for series (unless you find it helpful).

Good Luck!

Score	
1	/10
2	/22
3	/22
4	/16
5	/12
6	/10
7	/12
Total	/100

1. (10 pts). Find all values $\tan^{-1}(-3i)$. [Write your final answer in the form $a+bi$.]

2. (22 pts). For each of the following integrals,

(*i*). Determine and ***briefly*** explain whether each of the integrals is independent of path.

(*ii*). If it is independent of path, evaluate the integral by finding an antiderivative. If it is not independent of path, then evaluate the integral by parameterization.

[You do NOT need to simplify to the form $a+bi$.]

(a). $\displaystyle\int_C \frac{3}{(z-1)^2}\,dz$ where C is line segment connecting $z=2$ and $z=2i$.

(b). $\displaystyle\int_C |z|^2\,dz$ where C is the curve $x=t^2, y=\dfrac{1}{t}$ for $1 \le t \le 2$

3. (22 pts). Use the Cauchy Integral Formula(s) to evaluate

(a). $\oint_C \frac{3-z^2}{z^2+2iz}\,dz$ where C is the circle $|z+2i|=1$.

(b). $\oint_C \frac{e^{z^2}}{(z-i)^3}\,dz$ where C is the circle $|z|=3$

4. (16 pts). Determine whether the following statements are GUARANTEED TO ALWAYS BE TRUE and give a ***brief*** explanation of your answer.

(a). $\int_{C_1} f(z)\,dz = \int_{C_2} f(z)dz = 0$ where C_1 is the square with vertices at $z = \pm 10 \pm 10i$ and C_2 is the circle $|z-(i+1)| = 2$, both oriented in the counterclockwise direction. Also $f(z)$ is analytic on C_1 and C_2 and the region in between both curves.

(b). $\int_C \frac{(5+i)z}{(z-2i)(z+2i)}\,dz = 0$ where C is the circle $|z|=1$.

(c). $\int_C \text{Log}\,(z+2)\,dz$ where C is the circle $|z|=1$.

(d). $\oint_C \frac{1}{\sin z}\,dz = \oint_{C_1} \frac{1}{\sin z}\,dz + \oint_{C_2} + \frac{1}{\sin z}\,dz + \oint_{C_3} \frac{1}{\sin z}\,dz$ where C is the rectangle with vertices at $\pm 5 \pm 2i$ and C_1, C_2, C_3 are the circles $|z+\pi| = 1, |z| = 1, |z-\pi| = 1$, respectively, all oriented in the counterclockwise direction.

5. (12 pts). Find an upper bound on $\left|\int_C \frac{e^z}{z^4+4z^2+4}\,dz\right|$ where C is the circle $z = 3e^{i\theta}$.

6. (10 pts). Determine whether the following series converges or diverges. If it converges, find the sum.

$$\sum_{n=0}^{\infty} \frac{2i^{1-n}}{(2-i)^n}$$

7. (12 pts). Given $f(z) = \frac{3}{8z^2+z^5}$

(a). Use a known Maclaurin Series to find a series expansion for $f(z)$ and state the values of z where it converges. [Write your answer so that all terms are within the summation and simplified. But do NOT shift any indices.]

(b). Is the resulting series a Maclaurin Series? Why or why not?