1. Evaluate the following and write your answer in the form $a+ib$.

(a). $\left(\frac{1}{2}-\frac{1}{4}i\right)\left(\frac{2}{3}+\frac{5}{3}i\right)=\frac{3}{4}+\frac{2}{3}i$

(b). $\frac{5}{i}+\frac{2}{i^3}-\frac{20}{i^{18}}=20-3i$

(c). $\frac{(4+5i)+2i^3}{(2+i)^2}=\frac{24}{25}-\frac{7}{25}i$

(d). $\frac{1}{i-3}=-\frac{3}{10}-\frac{1}{10}i$

2. Solve the following equations for $z=a+ib$.

(a). $2z=i(2+9i)$ $\quad z=-\frac{9}{2}+i$

(b). $\frac{z}{1+\bar{z}}=3+4i$ $\quad z=-\frac{7}{6}-\frac{1}{6}i$

3. Describe or sketch the set of points z in the complex plane that satisfy the following equations.

$\text{Re}\,(z^2)=|\sqrt{3}-i|$ hyperbola: $x^2-y^2=2$

4. Find an upper bound on $\frac{1}{|z^4-5z^2+6|}$ if $|z|=2$. $\quad |z^4-5z^2+6|\geq ||z^4|-|5z^2|-|6|| \Longrightarrow \frac{1}{|z^4-5z^2+6|}\leq\frac{1}{10}$

5. Verify that $\sqrt{2}|z|\geq|\text{Re}\,z|+|\text{Im}\,z|$ by reducing it to the inequality $(|x|-|y|)^2\geq 0$ and explain why this is true.

6. Write the given complex number in polar form, first using $\theta\neq\text{Arg}(z)$ and then using $\theta=\text{Arg}(z)$.

$-2-2\sqrt{3}i$

7. Calculate the following by first converting to polar form. Then write your final answer back in rectangular form $a+ib$.

(a). $\frac{-i}{1+i}$

(b). $(1+\sqrt{3}i)^9$

8. Are there any cases in which $\text{Arg}\,(z_1z_2)=\text{Arg}\,z_1+\text{Arg}\,z_2$). Can you prove you assumptions? While this is true for $-\pi<\text{Arg}\,z_1+\text{Arg}\,z_2\leq\pi$, you can prove it specifically if $\text{Re}\,z_1>0$ and $\text{Re}\,z_2>0$.

9. Compute all of the roots, sketch them graphically, and indicate the principal root.

$$(1+i)^{1/5}$$

10. Sketch the set S.

(a). $\text{Re}\,(z^2)>0$

(b). $|z-i|>0$

(c). $0\leq\arg(z)\leq\pi/6$

(a). Sketch lines $y=\pm x$ and test regions

(b). Entire plane with a hole at i

(c). Wedge of plane between the angles

11. Find the natural domain of the function $\dfrac{3z+2i}{z^3+4z^2+z}$

12. Find the real and imaginary parts u and v of f both as functions of x and y (i.e. $f(z) = u(x,y) + iv(x,y)$) and of functions of r and θ (i.e. $f(z) = u(r,\theta) + iv(r,\theta)$).

$f(z) = z + \dfrac{1}{z}$

$f(z) = (x + \frac{x}{x^2+y^2}) + i(y - \frac{y}{x^2+y^2})$ and $f(z) = \left(r + \frac{1}{r}\right)\cos\theta + i\left(r - \frac{1}{r}\right)\sin\theta$

13. Show that $|e^z| = e^x$

14. Show that $\displaystyle\lim_{z\to 0} \frac{\text{Re } z}{\text{Im } z}$ does not exist

15. Verify the following limits. Justify your answers using the the theorem in Section 17.

(a). $\displaystyle\lim_{z\to\infty} \frac{z^2+iz-2}{(1+2i)z} = \frac{1}{5} - \frac{2}{5}i$ **(b).** $\displaystyle\lim_{z\to\infty} \frac{z^2-3iz+1}{iz+2} = \infty$ **(c).** $\displaystyle\lim_{z\to i} \frac{z^2-1}{z^2+1} = \infty$

(a). Theorem, part (2) **(b).** Theorem, part (3) **(c).** Theorem, part (1)

16. Determine where the following functions are differentiable and then find $f'(z)$.

(a). $f(z) = 4x^2 + 5x - 4y^2 + 9 + i(8xy + 5y - 1)$ **(b).** $f(z) = \dfrac{x}{x^2+y^2} + i\dfrac{y}{x^2+y^2}$

17. Find $f'(z)$

(a). $f(z) = z - \dfrac{1}{z}$ **(b).** $f(z) = (iz^3 - 7z^2)(z^3 - 3z^2)^4$

18. Find real constants, a and b so that the following function is analytic.
$f(z) = 3x - y + 5 + i(ax + by - 3)$ $a = 1, b = 3$

19. Express each of the following in the form $a + ib$.

(a). $\sin(-3i) = -i\sinh 3$

(b). $\cos(2-4i) = \cos 2\cosh 4 + i\sin 2\sinh 4$

(c). $\tan(\frac{\pi}{2} - i) = i\coth(1)$

(d). $\cosh\left(1 + \frac{\pi}{6}i\right) = \frac{1}{2}\cosh(1)\sqrt{3} + \frac{1}{2}i\sinh(1)$

(e). $\tanh \pi i = 0$

20. Find all values of z that satisfy the given equation. [You may find it helpful to equate real and imaginary parts.]

(a). $\cos z = i\sin z$ **(b).** $\sin z = i$ **(c).** $\sinh z = i$ **(d).** $\cos z = \sinh 3$

21. Prove the following:

(a). $\overline{\cos z} = \cos \bar{z}$ **(b).** $|\sinh z|^2 = \sinh^2 x + \sin^2 y$ **(c).** $\tanh(z + \pi i) = \tanh z$

22. Find the derivatives

(a). $\sin z \cosh z$ **(b).** $\tanh(iz - 2)$

23. If $|\sin z| \leq 1$, then what can you say about z. Justify your answer.

z must be real, otherwise the magnitude is unbounded because of the $\sinh y$ term.

24. Find ***all*** complex values and state the Principle Value of each of the following.

(a). $\log(-2 + 2i)$

(b). $\log(ei)$

(c). $\log(1 + i\sqrt{3})$

(d). $(-1 - i)^{(3i)}$

(e). 2^{4i}

25. page 100: #1

26. Given $f(z) = \dfrac{\text{Log }(2z + i)}{z^2 + 4}$.

(a). Determine where the function is analytic.

(b). Find $f'(z)$.

27. Verify that $(z^\alpha)^n = z^{n\alpha}$ for $z \neq 0$ and n an integer.