1. Evaluate the following and write your answer in the form $a + ib$.

(a). $\left(\frac{1}{2} - \frac{1}{4}i\right)\left(\frac{2}{3} + \frac{5}{3}i\right)$

(b). $\frac{5}{i} + \frac{2}{i^3} - \frac{20}{i^{18}}$

(c). $\frac{(4+5i)+2i^3}{(2+i)^2}$

(d). $\frac{1}{i-3}$

2. Solve the following equations for $z = a + ib$.

(a). $2z = i(2+9i)$

(b). $\frac{z}{1+\bar{z}} = 3 + 4i$

3. Describe or sketch the set of points z in the complex plane that satisfy the following equations.

$\text{Re}\,(z^2) = |\sqrt{3} - i|$

4. Find an upper bound on $\frac{1}{|z^4 - 5z^2 + 6|}$ if $|z| = 2$.

5. Verify that $\sqrt{2}|z| \geq |\text{Re}\, z| + |\text{Im}\, z|$ by reducing it to the inequality $(|x| - |y|)^2 \geq 0$ and explain why this is true.

6. Write the given complex number in polar form, first using $\theta \neq \text{Arg}(z)$ and then using $\theta = \text{Arg}(z)$.

$-2 - 2\sqrt{3}i$

7. Calculate the following by first converting to polar form. Then write your final answer back in rectangular form $a + ib$.

(a). $\frac{-i}{1+i}$

(b). $(1 + \sqrt{3}i)^9$

8. Are there any cases in which $\text{Arg}\,(z_1 z_2) = \text{Arg}\, z_1 + \text{Arg}\, z_2)$. Can you prove you assumptions?

9. Compute all of the roots, sketch them graphically, and indicate the principal root.

$$(1+i)^{1/5}$$

10. Sketch the set S.

(a). $\text{Re}\,(z^2) > 0$

(b). $|z - i| > 0$

(c). $0 \leq \arg(z) \leq \pi/6$

11. Find the natural domain of the function $\dfrac{3z+2i}{z^3+4z^2+z}$

12. Find the real and imaginary parts u and v of f both as functions of x and y (i.e.$f(z) = u(x,y) + iv(x,y)$) and of functions of r and θ (i.e.$f(z) = u(r,\theta) + iv(r,\theta)$).
$f(z) = z + \dfrac{1}{z}$

13. Show that $|e^z| = e^x$

14. Show that $\displaystyle\lim_{z\to 0} \frac{\text{Re } z}{\text{Im } z}$ does not exist

15. Verify the following limits. Justify your answers using the the theorem in Section 17.

(a). $\displaystyle\lim_{z\to\infty} \frac{z^2+iz-2}{(1+2i)z} = \frac{1}{5} - \frac{2}{5}i$ **(b).** $\displaystyle\lim_{z\to\infty} \frac{z^2-3iz+1}{iz+2} = \infty$ **(c).** $\displaystyle\lim_{z\to i} \frac{z^2-1}{z^2+1} = \infty$

16. Determine where the following functions are differentiable and then find $f'(z)$.

(a). $f(z) = 4x^2 + 5x - 4y^2 + 9 + i(8xy + 5y - 1)$ **(b).** $f(z) = \dfrac{x}{x^2+y^2} + i\dfrac{y}{x^2+y^2}$

17. Find $f'(z)$

(a). $f(z) = z - \dfrac{1}{z}$ **(b).** $f(z) = (iz^3 - 7z^2)(z^3 - 3z^2)^4$

18. Find real constants, a and b so that the following function is analytic.
$f(z) = 3x - y + 5 + i(ax + by - 3)$

19. Express each of the following in the form $a + ib$.

(a). $\sin(-3i)$

(b). $\cos(2-4i)$

(c). $\tan(\frac{\pi}{2} - i)$

(d). $\cosh\left(1 + \frac{\pi}{6}i\right)$

(e). $\tanh \pi i$

20. Find all values of z that satisfy the given equation. [You may find it helpful to equate real and imaginary parts.]

(a). $\cos z = i \sin z$ **(b).** $\sin z = i$ **(c).** $\sinh z = i$ **(d).** $\cos z = \sinh 3$

21. Prove the following:

(a). $\overline{\cos z} = \cos \bar{z}$ **(b).** $|\sinh z|^2 = \sinh^2 x + \sin^2 y$ **(c).** $\tanh(z + \pi i) = \tanh z$

22. Find the derivatives

(a). $\sin z \cosh z$ **(b).** $\tanh(iz - 2)$

23. If $|\sin z| \leq 1$, then what can you say about z. Justify your answer.

24. Find ***all*** complex values and state the Principle Value of each of the following.

(a). $\log(-2 + 2i)$

(b). $\log(ei)$

(c). $\log(1 + i\sqrt{3})$

(d). $(-1 - i)^{(3i)}$

(e). 2^{4i}

25. page 100: #1

26. Given $f(z) = \dfrac{\text{Log}\,(2z + i)}{z^2 + 4}$.

(a). Determine where the function is analytic.

(b). Find $f'(z)$.

27. Verify that $(z^\alpha)^n = z^{n\alpha}$ for $z \neq 0$ and n an integer.