Name: _____ **Exam 1**

Math 434, Complex Variables – Crawford **13 March 2015**

- Books and notes (in any form) are not allowed.
- You may use calculators and the provided formula sheet.
- Put all of your work and answers other paper. Include this sheet as a cover sheet.
- ***Show all your work***. Partial credit may be given for written work.
- Unless otherwise stated,
  - ***Brief*** explanations should only require 1-2 sentences.
  - Simplify/Evaluate trigonometric, exponential, logarithmic, and hyperbolic functions for standard values.

Good Luck and Happy Pi Day Tomorrow!

	Score
1	/30
2	/8
3	/8
4	/12
5	/12
6	/10
7	/12
8	/12
Total	/100

1. (30 pts). Evaluate the following. [If it is multiple-valued, find all values.]

(a). $\dfrac{1}{i} - \dfrac{2-4i}{(3+i)^2}$ [Show intermediate steps and write in the form $a+ib$.]

(b). $(-2+2\sqrt{3}i)^{1/4}$ [Compute all roots and write them in the form $a+bi$, sketch them graphically, and indicate the principal root.]

(c). $(-2+2\sqrt{3}i)^{-i}$ [Write your answer in polar form $re^{i\theta}$. Sketch or describe the points in the complex plane.]

2. (8 pts). Describe or sketch the set of points z in the complex plane that satisfy the equation $|z+2i| = \text{Im}\,(z)$.

3. (8 pts). Use the theorems from class about limits involving infinity to show that

$\lim\limits_{z\to\infty} \dfrac{2iz^2+3}{(i+1)z} = \infty$ [Show all steps for the theorem used.]

4. (12 pts). Use the limit definition of the derivative $f'(z) = \lim\limits_{\Delta z\to 0} \dfrac{f(z+\Delta z)-f(z)}{\Delta z}$ to show that

$f(z) = \text{Re}\,(z) = x$ is <u>not</u> differentiable anywhere.

5. (12 pts). Given $f(z) = \sqrt{r}e^{i\theta/2}$ $(r>0, -\pi<\theta<\pi)$,

(a). Use the Cauchy-Riemann equations to show that $f(z)$ is differentiable.

(b). Find $f'(z)$. [You do not need to simplify.]

6. (10 pts). Determine where the following function is not analytic <u>and</u> sketch the resulting branch cut.

$f(z) = \text{Log}\,(iz-3)$

7. (12 pts). Given the branch $\log z = \ln r + i\theta$ $\left(r>0, -\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{2}\right)$, determine <u>whether</u>

$\log(1+i)^3 = 3\log(1+i)$. [Show all your work to justify your answer.]

8. (12 pts). Find all values of z that satisfy the given equation. $\cos z = 3i$

[You may leave inverse trig and/or inverse hyperbolic functions in your answer(s).]