THEOREM If $X_1, X_2, \ldots, X_n$ are n mutually independent ____________ random variables with means $\mu_1, \mu_2, \ldots, \mu_n$ and variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$,

then $Y = \sum c_i X_i = c_1X_1 + c_2X_2 + \ldots + c_nX_n$ has the normal distribution ______________

where $\mu_y = \sum c_i\mu_i$ and $\sigma_y^2 = \sum c_i^2\sigma_i^2$.

COROLLARY Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$.

Then the sample mean $\overline{X} = \dfrac{1}{n}\sum X_i = \dfrac{X_1 + X_2 + \ldots + X_n}{n}$ has the normal distribution _______________ .

So $\overline{X}$ has the same mean, but a smaller variance. Hence, the probability that $\overline{X}$ falls in a given interval containing μ is greater than the probability that a single observation of X falls in the same interval.

EX Let $X_1, X_2, \ldots, X_{10}$ be a random sample from test scores distributed as $N(76.4, 4.1^2)$.

(a). Find $P(70 < X_1 < 80)$.

(b). Find $P(70 < \overline{X} < 80)$.

Let $W = \frac{\sqrt{n}}{\sigma}(\overline{X} - \mu) = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$ where X comes from __________ distribution.

Then $\mu_W = E(W) = E\left(\frac{\overline{X} - \mu}{\sigma/\sqrt{n}}\right) =$

and

$\sigma_W^2 = \text{Var}(W) = E(W^2) - (E(W))^2 = E(W^2) - 0^2$

So W has mean ________ and variance ________ regardless of ____
.

But how is W itself distributed?

<u>Central Limit Theorem</u> If $\overline{X}$ is the mean of a random sample $X_1, X_2, \ldots, X_n$ of size n from a distribution with finite mean μ and finite variance σ^2, then the distribution of

$W = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$ is

Comments

Ex Let $X_1, X_2, \ldots, X_{30}$ be a random sample from an unknown distribution with mean 28.5 and variance 9.8.

(a). Find $P(28 < \overline{X} < 30)$

(b). Let $Y = X_1 + X_2 + \ldots + X_{30}$. Find $P(800 < Y < 900)$

Ex Let $X_1, X_2, \ldots, X_{25}$ be a random sample from a $\chi^2(2)$. Let $Y = X_1 + X_2 + \ldots + X_{25}$.

(a). Find $P(45 < Y < 55)$ using the distribution of Y.

(b). Find $P(45 < Y < 55)$ using the Central Limit Theorem.