All the definitions and results concerning two random variables of the discrete type can be carried over to two random variables of the continuous type. Moreover, the notions about two independent random variables can be extended to n independent random variables, which can be thought of as measurements on the outcomes of n random experiments. That is, if $X_1, X_2, \ldots, X_n$ are **independent**, then the joint pmf or pdf is the product of the respective pmfs or pdfs, namely, $f_1(x_1)f_2(x_2)\cdots f_n(x_n)$.

Ex: Let X_1, X_2, and X_3 be independent random variables with binomial distributions b(4,0.2), b(10,0.6), b(8,0.2).

Find $P(X_1 = 1, X_2 = 6, X_3 = 4)$

If all n of the distributions are the same, then the collection of n independent and identically distributed random variables, $X_1, X_2, \ldots, X_n$, is said to be a **random sample of size n from that common distribution**. If $f(x)$ is the common pmf or pdf of these n random variables, then the joint pmf or pdf is $f(x_1)f(x_2)\cdots f(x_n)$.

5.3-16. Each of eight bearings in a bearing assembly has a diameter (in millimeters) that has the pdf

$$f(x) = 10x^9, \qquad 0 < x < 1.$$

Assuming independence, find the cdf and the pdf of the maximum diameter (say, Y) of the eight bearings and compute $P(0.9999 < Y < 1)$.

Of course, when we are dealing with n random variables that are not independent, the joint pmf (or pdf) could be represented as

$$f(x_1, x_2, \ldots, x_n), \qquad (x_1, x_2, \ldots, x_n) \in S.$$

The **mathematical expectation** (or **expected value**) of $u(X_1, X_2, \ldots, X_n)$ is given by

$$E[u(X_1, X_2, \ldots, X_n)] = \sum \sum_{S} \cdots \sum u(x_1, x_2, \ldots, x_n) f(x_1, x_2, \ldots, x_n).$$

Of course, $Y = u(X_1, X_2, \ldots, X_n)$ has a distribution with a pmf (or pdf), say $g(y)$. It is true, but we do not prove it, that

$$E(Y) = \sum_{y} y\, g(y) = \sum \sum_{S} \cdots \sum u(x_1, x_2, \ldots, x_n) f(x_1, x_2, \ldots, x_n).$$

Theorem 5.3-1 Say $X_1, X_2, \ldots, X_n$ are n independent random variables and $Y = u_1(X_1)\, u_2(X_2) \cdots u_n(X_n)$. If $E[u_i(X_i)],\ i = 1, 2, \ldots, n$, exist, then

$$E(Y) = E[u_1(X_1)\, u_2(X_2) \cdots u_n(X_n)] = E[u_1(X_1)]E[u_2(X_2)] \cdots E[u_n(X_n)].$$

(See book for proof in Discrete Case)

Theorem 5.3-2 If $X_1, X_2, \ldots, X_n$ are n random variables with respective means $\mu_1, \mu_2, \ldots, \mu_n$ and variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$, then the mean of $Y = \sum_{i=1}^{n} a_i X_i$, where $a_1, a_2, \ldots, a_n$ are real constants, is

$$\mu_Y = \sum_{i=1}^{n} a_i \mu_i.$$

If, in addition, $X_1, X_2, \ldots, X_n$ are independent, then the variance of Y is

$$\sigma_Y^2 = \sum_{i=1}^{n} a_i^2 \sigma_i^2.$$

EX. Let X_1, X_2, and X_3 be independent random variables that represent lifetimes (in hours) of 3 key components of a device. Their representative distributions are exponential with means 1000, 1500, and 2000, respectively.

a) Find the joint pdf of X_1, X_2, and X_3
b) Find $P(0< X_1<500,\ 0<X_2<750,\ 0<X_3<1000)$
c) Let $Y = X_1 + X_2 + X_3$. Find the mean and variance of Y.
d) Find $E(\overline{X})$ and $Var(\overline{X})$

Now consider the **mean of a random sample**, $X_1, X_2, \dots, X_n$, from a distribution with mean μ and variance σ^2, namely,

$$\overline{X} = \frac{X_1 + X_2 + \cdots + X_n}{n},$$

which is a linear function with each $a_i = 1/n$. Then

$$\mu_{\overline{X}} = \sum_{i=1}^{n} \left(\frac{1}{n}\right)\mu = \mu \qquad \text{and} \qquad \sigma_{\overline{X}}^2 = \sum_{i=1}^{n} \left(\frac{1}{n}\right)^2 \sigma^2 = \frac{\sigma^2}{n}.$$

EX. For each i = 1, 2, 3…, n, the random variable X_i is $N(100,36)$. Find the distribution of $\bar{X}$ and illustrate it for n = 3, 4, 6, and 12.

HW: Section 5.3, p. 186: #1, 3, 7, 10, 13, 17[Hint: Not Indep.], 18, 19