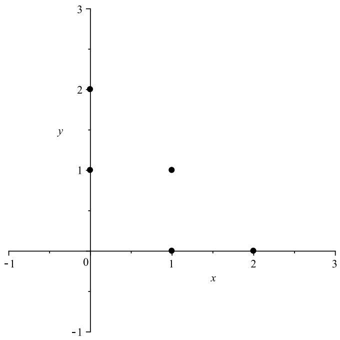
Given $f(x) = \dfrac{x+y}{8}$, for $1 \le x+y \le 2$ and x, y nonnegative integers,



(a). Sketch the support of X and Y and probabilities on the graph above.

(b). Add the marginal p.m.f.'s $f_1(x)$ and $f_2(y)$ to the graph.

(c). Find μ_X and μ_Y and sketch the point (μ_X, μ_Y) on the graph above.

$$\mu_X = E[X] = \sum_x x \cdot f_1(x) = 0 \cdot \frac{3}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{1}{4} = \frac{7}{8}$$

$$\mu_Y = E[Y] = \sum_y y \cdot f_2(y) = 0 \cdot \frac{3}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{1}{4} = \frac{7}{8}$$

(d). Find the $\text{Cov}(X, Y)$.

$$E[XY] = \sum\sum_{(x,y)\in S} xyf(x,y) = 1 \cdot 0 \cdot \frac{1}{8} + 2 \cdot 0 \cdot \frac{1}{4} + 0 \cdot 1 \cdot \frac{1}{8} + 1 \cdot 1 \cdot \frac{1}{4} + 0 \cdot 2 \cdot \frac{1}{4} = \frac{1}{4}$$

$$\sigma^2_{XY} = \text{Cov}(X, Y) =$$

(e). Find the correlation coefficient ρ.

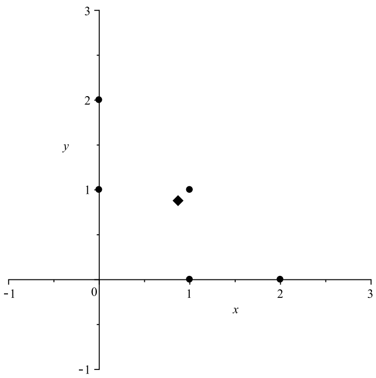
$$E[X^2] = \sum_x x^2 \cdot f_1(x) = 0^2 \cdot \frac{3}{8} + 1^2 \cdot \frac{3}{8} + 2^2 \cdot \frac{1}{4} = \frac{11}{8}$$

$$E[Y^2] = \sum_y y^2 \cdot f_2(y) = 0^2 \cdot \frac{3}{8} + 1^2 \cdot \frac{3}{8} + 2^2 \cdot \frac{1}{4} = \frac{11}{8}$$

$$\sigma^2_X = E[X^2] - (E[X])^2 = \frac{11}{8} - \left(\frac{7}{8}\right)^2 = \frac{11}{8} - \frac{49}{64} = \frac{88-49}{64} = \frac{39}{64}$$

$$\sigma^2_Y = E[Y^2] - (E[Y])^2 = \frac{11}{8} - \left(\frac{7}{8}\right)^2 = \frac{11}{8} - \frac{49}{64} = \frac{88-49}{64} = \frac{39}{64}$$

$$\rho =$$



(a). Draw a line through (μ_X, μ_Y) with any slope b that seems to reasonable with the data.

(b). Write down the equation for this line.

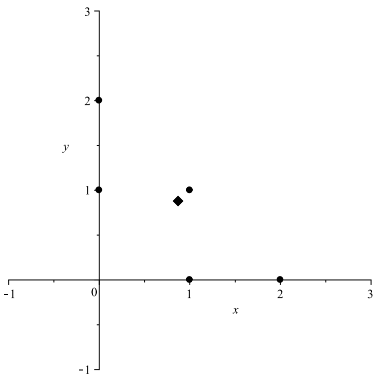
(c). Write an expression for the vertical distance D between a point (x_0, y_0) in the support and the vertical line.

(d). Find the expected value (or weighted mean) of D^2.

(e). Find the slope b that minimizes this expected value (mean of squared distance).

$$\begin{aligned} K(b) &= E[((Y-\mu_Y)-b(X-\mu_X))^2] \\ &= E[(Y-\mu_Y)^2-2b(X-\mu_x)(Y-\mu_Y)+b^2(X-\mu_x)^2] \\ &= \end{aligned}$$

Find the Least-Squares Line for the example from page 1. Sketch it on the graph.



At the optimal b value:

$E[\text{squared distance } D^2] = K(b) = K\left(\rho \frac{\sigma_Y}{\sigma_X}\right) = work =$

Homework: Section 4.2, p. 144: #1, 3, 9, 10,11