Examples of events that follow a Poisson Distribution:

- The number of cars passing a specific light during rush hour
- The number of customers arriving at a checkout line each minute
- The number of missprints per page of a manuscript
- The number of a certain species of a plant per square foot area

1. Ex Suppose one page of a manuscript has, on average, 2.6 misprints. The misprints are considered to have a poisson distribution. If one page is selected at random, find the following probabilities for the number of misprints on that page.

(a). 0 misprints

(b). 3 misprints

(c). at most 3 misprints

(d). at most 8 misprints

(e). Suppose each chapter is approximately 15 pages. What is the probability of having 30 misprints in a chapter?

2. Ex An actuary finds that policy holders are 4 times as likely to file 2 claims as to file 3 claims in a year. If the number of claims filed follows a Poisson distribution, find the variance of the number of claims.

3. Ex A baseball team schedules its opening game for April 1. If it rains on April 1, the game is postponed and played the next day it does not rain. The team purchases an insurance policy that pays \$1000 each day, up to 2 days, that the opening game is delayed by rain. Suppose the number of consecutive days of rain beginning April 1 has a poisson distribution with $\lambda = 0.6$.

(a). Find the expected value of the payment before the opening game.

(b). Find the standard deviation of the payment.

(c). How would the expected value of the payment change if \$1000 paid for each rain delay day, with no limit on number of days?

<u>Def</u> Let the number of times an event occurs be counted. Let $\lambda > 0$ be the mean rate of occurrence per unit period. An (approximate) Poisson Process with parameter λ must satisfy the following properties.

(a). The number of occurrences in ____________________ (of sufficiently short length h) are independent.

(b). The probability of exactly one occurrence in a sufficiently short interval of length h is ______ .

(c). The probability of 2 or more occurrences in a sufficiently short interval of length h is ______ .

Let X be the number of occurrences in an interval of ______________ for an experiment that satisfies the conditions of a Poisson Process.

Find the p.m.f $f(x)$ for X. i.e. Find

[Start w/ an approximation ________________ .]

Divide the interval into n subintervals of length ______

- By property (b), the probability of exactly one occurrence in each subinterval is ____________ .
- By property (c), the probability of 2 or more occurrences in each subinterval is ______ .
- By property (a), the number of occurrences in each of the n subintervals is ____________ .

Let Success = 1 represent an occurrence in a subinterval.

Let Failure = 0 represent no occurrence in a subinterval.

Then the probability of success is $p =$ ______ and the probability of failure is $q =$ ________ .

n subintervals with ____________ occurrences $\Rightarrow$ n independent trials $\Rightarrow$ n Bernoulli trials

$\Rightarrow$ ________________________ is an ***approximation*** for __________ (i.e. the p.m.f of X).

For $X = x$, the value x counts the number of occurrences in an interval of length ________ . [Poisson]

This value of x is approximated by counting the number of subintervals of length ________ that have

__________________________ . [Binomial]

[Sketch picture]

So for the Poisson Distribution $f(x) = P(X = x) \approx \text{binom}\left(n, \frac{\lambda}{n}\right) =$

Take the limit to get the exact p.m.f for the Poisson Distribution.

$$f(x) = P(X = x) = \lim_{n\to\infty} \frac{n!}{x!(n-x)!}\left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

Homework: Section 2.7, p. 86: #1, 3, 4, 5, 7, 9, 11, 12, 13,