Ex A class has 18 students, 10 of whom are female and 8 are male. A group of 4 students is randomly selected. Let X be the number of females in the group. Find the pmf of X.

$P(X = 0) =$

$P(X = 1) =$

$P(X = 2) =$

$P(X = 3) =$

$P(X = 4) =$

Collection of $N =$ ______________ objects

_________ of type 1 and _________ of type 2

Select ________ objects at random ________________________

Find the probability that exactly x of n objects are of type 1.

$f(x) =$

$\mu =$

$\sigma^2 =$

Ex Same example.

(a). Find the mean and variance.

(b). Find the probability that at least 1 female is in the group of 4.

(c). Find the probabiilty that at most 2 females are in the group of 4.

Ex Suppose X is the number of Bernoulli trials until the r^{th} success is observed.

Let $x =$ the trial number where the r^{th} success occurred.

Then the __________ trials must have exactly __________ successes: $SSS \ldots SFFF \ldots F$

$\Longrightarrow$ ways to get $r-1$ successes in $x-1$ trials. Each of which has the probability ____________

So the first $x-1$ trials have a probability given by ____________________ .

Then the last (x^{th}) trial must be a success with probability ______ .

So the pmf for X is $g(x) = \binom{x-1}{r-1} p^{r-1} q^{x-r}$.

Negative Binomial Distribution

where x is the number of trials until the r^{th} success is observed.

Given without proof (yet): $\mu = \dfrac{r}{p}$ and $\sigma^2 = \dfrac{rq}{p^2} = \dfrac{r(1-p)}{p^2}$

Ex A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents occur in a given month is $\frac{7}{10}$. Consider accidents occurring each month independent from each other. Find the following probabilities.

(a). First accident occurs in the 4th month.

(b). The 12th month is the 3rd month with accidents that year.

Part(a) is a special case when $r = 1$:

Let X be the number of trials until the 1^{st} success.

Then $g(x) = \binom{x-1}{0} p^1 q^{x-1} = pq^{x-1}$

Geometric Distribution

Then $\mu = \frac{1}{p}$ and $\sigma^2 = \frac{q}{p^2} = \frac{(1-p)}{p^2}$

Note: $\mu = \frac{1}{p}$ is intuitive:

If the probability of success is $p = \frac{1}{10}$, then we expect, on average, $10 = \frac{1}{1/10}$ trials before the 1^{st} success.

For the Geometric Distribution, find $P(X > k)$ for integer k.

$$P(X > k) = \qquad = pq^{(k+1)-1} + pq^{(k+2)-1} + \cdots =$$

$$= \sum_{x=k+1}^{\infty} pq^{-1}q^x =$$

$$= pq^{-1} \cdot \qquad = p \cdot \frac{q^k}{1-q}$$

$$= p \cdot \frac{q^k}{\quad}$$

$$= q^k = (1-p)^k$$

i.e. and

Ex Consider the previous example and find the following probabilities.

(a). More than 3 months go by without an accident.

(b). At most 3 months go by before the first accident.

Homework: Section 2.5, p. 75: #3, 5, 9; Section 2.6, p. 80: #1, 3, 4, 9