

- 1. How many possible ways are there to rearrange the letters in the word SUM?

- 2. How many possible ways are there to rearrange the letters in the word ADD?

- 3. How many possible ways are there to rearrange the letters in the word DADDA?

Permutation Rule (some items identical)

If you have a set of n items in which there are duplicate items, then the total number of ways to rearrange all n items into _____ is given by

n_1 = the number of duplications of item 1,
 n_2 = the number of duplications of item 2,
⋮
 n_s = the number of duplications of item s ,
and
 $n_1 + n_2 + \dots + n_s = n$.

4. Given an organization with 12 members,

(a). How many different ways could they choose a President, Vice-President, and Treasurer?

(b). If they need to send a delegation of 3 members to a conference, how many different delegations could be formed?

Def. Given n objects and choosing r of them where order does not matter, the resulting unordered subsets are called _____ . The total number of _____ is given by _____

Note: ${}_nC_r = \binom{n}{r}$ are also called the _____

$$(x+y)^4 = \sum_{r=0}^4 \binom{4}{r} x^{4-r} y^r = \binom{4}{0} x^4 y^0 + \binom{4}{1} x^3 y^1 + \binom{4}{2} x^2 y^2 + \binom{4}{3} x^1 y^3 + \binom{4}{4} x^0 y^4 = x^4 + 4x^3 y + 6x^2 y^2 + 4xy^3 + y^4.$$

Similarly, $\binom{n}{n_1, n_2, \dots, n_s} = \frac{n!}{n_1! n_2! \dots n_s!}$ is called the _____ because it gives the coefficient of the term $x_1^{n_1} x_2^{n_2} \dots x_s^{n_s}$ in the multinomial expansion $(x_1 + x_2 + \dots + x_s)^n$

5. Suppose a study is conducted of families with 5 children. List out all possible ways of having 3 girls and 2 boys.

Is it a coincidence that this number ended up being _____ ?

Suppose a set of n objects has r of one type and $n - r$ (the rest) of another type. The number of distinguishable permutations is given by

EXAMPLES

1. An organization with 16 members, 9 male and 7 female, must form a team of 4 members. The team must have 2 males and 2 females. How many ways can such a team be formed?

2. An automobile comes in 8 possible colors. You also have 5 possible upgrades. How many different cars are possible if you choose

(a). 2 upgrades?

(b). 0, 1, up to 5 upgrades?

(c). 0, 1, or 2 upgrades?

3. In the Illinois Little Lotto, a person chooses 5 numbers between 1 and 39 without replacement. The first three prizes are determined as follows.

First Prize: Match all 5 numbers in any order and win \$100,000.

Second Prize: Match 4 numbers in any order and win \$100.

Third Prize: Match 3 numbers in any order and win \$10.

Find the probability of winning each of these prizes.

4. Given a standard 52-card deck, find the probability of getting the following 5 card hands.

(a). Pair of Aces with a pair of Kings.

(b). Three of a kind.