**Name:** _____ **Take-Home Quiz 2**

**Math 451 Probability Theory – Crawford** **Due: Friday, November 8, 2019 by 10:00 am**

This is a take-home quiz. You ***are*** allowed to work with each other, but you may not get help from me. Put all of your work and answers on your own paper. Attach this sheet as a cover sheet. You must *show all your work* for full credit. Good Luck! [Scores will be scaled to 20 points.]

**1.** (12 pts) In an experiment to measure the intelligence of mice, the time it takes for them to go through a maze to reach a food reward is measured. The time (in seconds) required is a random variable $X$ with a density function given by

$$f(x) = \begin{cases} \dfrac{m}{x^2}, & x \geq m \\ \\ 0, & \text{otherwise} \end{cases}$$

where $m$ is the minimum time required to complete the maze.

**(a)**. Show that $f(x)$ has the properties of a density function.

**(b)**. Find a formula for the cdf $F(x)$ and sketch $F(x)$.

**(c)**. If $c$ and $d$ are both positive constants such that $d > c$ ,find $P(X > m + d | X > m + c)$.

**(d)**. Find the expected value of $X$.

**2.** (9 pts) The length of time to failure (in hundreds of hours) for a transistor is a random variable $X$ with distribution function given by

$$F(x) = \begin{cases} 0, & x < 0 \\ \\ 1 - e^{-x^2} & x \geq 0 \end{cases}$$

**(a)**. Find the first quartile (i.e., $q_1 = \pi_{.25}$).

**(b)**. Find the pdf $f(x)$.

**(c)**. Find $P(X > 100 | X \leq 200)$.

**3.** (6 pts) The magnitude of earthquakes recorded in a certain region of Japan can be modeled as having an exponential distribution with mean 3.1, as measured on the Richter scale. Find the probability that an earthquake striking this region will

**(a)**. Exceed 4.0 on the Richter scale.

**(b)**. Fall between 3.0 and 4.0 on the Richter scale.

**4.** (6 pts) The weekly amount of downtime $Y$ (in hours) for an industrial machine has approximately a gamma distribution with $\alpha = 3$ and $\theta = 2$.

**(a)**. Find the mean and variance of $Y$.

**(b)**. Find the probability that the weekly downtime will be greater than 4 hours.

**5.** (6 pts) A potato farmer creates 5 lb. bags of medium potatoes. The packaging for medium potatoes indicates that they are approximately 5.3 ounces. If the distribution of the weights for a batch of medium potatoes is $N(5.5, 0.14)$. Let $X$ denote the weight of a single potato selected at random from the this batch.

**(a)**. Find $P(X > 5.65)$.

**(b)**. Suppose that 12 potatoes are selected independently and weighed. Let $Y$ equal the number of these potatoes that weigh less than 5 ounces. Find $P(Y > 2)$.