Take Home Quiz 2 **Key**

1. $f(x) = \begin{cases} m/x^2, & x \geq m \\ 0, & \text{otherwise} \end{cases}$

a) $\int_{-\infty}^{\infty} f(x)dx = \int_m^{\infty} \frac{m}{x^2}dx = \int_m^{\infty} m x^{-2} dx = -m x^{-1} \Big|_m^{\infty}$

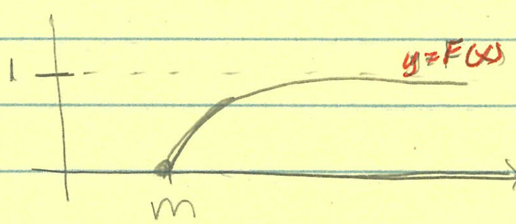
$= \lim_{x \to \infty} \underbrace{-\frac{m}{x}}_{0} + \cancel{\frac{m}{m}}^{1} = 1$ ✓

and $f(x) \geq 0 \quad \forall x \geq m$ ✓

b) $F(x) = P(X \leq x) = \int_m^x f(t)dt$

$= \int_m^x \frac{m}{t^2}dt = -\frac{m}{t}\Big|_m^x = -\frac{m}{x} + \frac{m}{m} = -\frac{m}{x} + 1$

ie $\boxed{F(x) = 1 - \frac{m}{x}, \quad x \geq m}$



c) $\overset{d>c}{P(X>m+d \mid X>m+c)} = \frac{P(X>m+d \text{ and } X>m+c)}{P(X>m+c)} = \frac{P(X>m+d)}{P(X>m+c)}$ (*)

$P(X>m+d) = 1 - P(X \leq m+d) = 1 - F(m+d) = 1 - \left(1 - \frac{m}{m+d}\right) = \frac{m}{m+d}$

$P(X>m+c) = 1 - P(X \leq m+c) = 1 - F(m+c) = 1 - \left(1 - \frac{m}{m+c}\right) = \frac{m}{m+c}$

From (*), $P(X>m+d \mid X>m+c) = \boxed{\frac{\left(\frac{m}{m+d}\right)}{\left(\frac{m}{m+c}\right)} = \frac{\cancel{m}}{m+d} \cdot \frac{m+c}{\cancel{m}} = \boxed{\frac{m+c}{m+d}}}$

(d) $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx = \int_m^{\infty} x \cdot \frac{m}{x^2}dx = \int_m^{\infty} \frac{m}{x}\, dx$

$= m \ln x \Big|_m^{\infty} = \lim_{x \to \infty} \overbrace{m \ln x}^{\infty} - m \ln m$

$\boxed{E(X) \text{ diverges to } \infty}$

2. $F(x) = \begin{cases} 0, & x < 0 \\ 1-e^{-x^2}, & x \geq 0 \end{cases}$

a) $q_1 = \pi_{.25}$: $P(X \leq q_1) = 0.25$

ie $F(q_1) = 0.25$

$\Rightarrow 1 - e^{-q_1^2} = .25$

$e^{-q_1^2} = .75 \Rightarrow -q_1^2 = \ln(.75)$

$q_1^2 = -\ln(.75)$

$q_1 = \sqrt{-\ln(.75)} \approx \boxed{0.5364}$ hundreds of hours.

b) $f(x) = F'(x)$

$$\boxed{f(x) = \begin{cases} 0, & x < 0 \\ 2xe^{-x^2}, & x \geq 0 \end{cases}}$$

oops should be $P(X>1 \mid X \leq 2)$

c) $P(X > 100 \mid X \leq 200) = \dfrac{P(X > 100 \text{ and } X \leq 200)}{P(X \leq 200)}$

$= \dfrac{P(100 < X \leq 200)}{P(X \leq 200)} = \dfrac{F(200) - F(100)}{F(200)} = \dfrac{1 - e^{-200^2} - (1 - e^{-100^2})}{1 - e^{-200^2}}$

$= \dfrac{e^{-100^2} - e^{-200^2}}{1 - e^{-200^2}} \approx \boxed{0}$

Corrected: $\dfrac{P(1 < X \leq 2)}{P(X \leq 2)} = \dfrac{F(2) - F(1)}{F(2)} = \dfrac{e^{-1^2} - e^{-2^2}}{1 - e^{-2^2}} \approx \boxed{0.3561}$

5. $N(5.5,\ 0.14)$

a) $P(X > 5.65) = \overset{\text{TI}}{\text{normalcdf}}(5.65,\ \overset{\text{est}}{\widetilde{9999}},\ 5.5,\ \sqrt{0.14})$

$\approx \boxed{.3442}$ TI-Calc.

OR $P(X > 5.65) = P\left(Z > \frac{5.65-5.5}{\sqrt{.14}}\right) = P(Z > .4009)$

$= 1 - P(Z < .4009)$

$\approx 1 - 0.6554$ Table Va

$\approx \boxed{0.3446}$

b) $n = 12$ potatoes $Y = $ # of potatoes that weigh less than 5oz

$p = P(X < 5) = \text{normalcdf}(-9999,\ 5,\ 5.5,\ \sqrt{.14})$

≈ 0.0907

$Y \sim \text{Binomial}(12,\ .0907)$

$P(Y > 2) = 1 - P(Y \le 2) = 1 - \text{binomcdf}(12,\ .0907,\ 2)$

$= 1 - .9118 \approx \boxed{.0882}$

OR $p = P(X < 5) = P\left(Z < \frac{5-5.5}{\sqrt{.14}}\right) = P(Z < -1.34)$

$= 1 - \Phi(1.34) = 1 - .9099 \approx .0901 = p$

$Y \sim \text{binomial}(12,\ .0901)$

$P(Y > 2) = 1 - P(Y \le 2) = 1 - P(Y=0) - P(Y=1) - P(Y=2)$

$= 1 - \frac{12!}{0!12!}(.0901)^0(.9099)^{12} - \frac{12!}{1!11!}(.0901)^1(.9099)^{11} - \frac{12!}{2!10!}(.0901)^2(.9099)^{10}$

$= 1 - (.9099)^{12} - 12(.0901)(.9099)^{11} - 66(.0901)^2(.9099)^{10}$

$\approx \boxed{.0869}$