Name: Key

Math 451 Probability Theory — Crawford

Quiz 1

12 September 2019

Books and notes (in any form) are not allowed. You may use a calculator. *Show all your work.* Good Luck!

**1.** (5 pts) Cards are dealt, one at a time, from a standard 52-card deck. If the first 3 cards are spades, what is the probability that the next 2 cards are also spades?

$$P(\text{Next 2 Spades}) = \frac{\binom{10}{2}}{\binom{49}{2}} = \frac{45}{1176} \approx \boxed{0.0383}$$

OR $\left(\frac{10}{49}\right)\cdot\left(\frac{9}{48}\right) \approx$

**2.** (5 pts) A study of 884 persons who received drug abuse treatment after a criminal conviction found that within a two-year period after the treatment, 327 of them had another conviction. Of the 884 persons, 353 had ten or more years of education. Of those 353 persons, 88 had another conviction within two years. If a single person is selected at random from the 884 people who received treatment, find the probability that

(a). The offender is convicted within two years of treatment given that the offender had less than 10 years of education.

Within 2 years

| Education | Conviction | No Conviction | |
|---|---|---|---|
| 10+ years | 88 | 265 | 353 |
| <10 years | 239 | 292 | 531 |
| | 327 | 557 | 884 |

Given

$$P(\text{Conviction} \mid <10\text{years}) = \frac{P(\text{Conviction} \cap <10\text{years})}{P(<10\text{ year})} = \boxed{\frac{239}{531} \approx 0.450}$$

(b). The offender has less than 10 years of eduction given that the offender is convicted within two years of treatment.

$$P(<10\text{years} \mid \text{convicted}) = \frac{P(<10\text{years} \cap \text{convicted})}{P(\text{convicted})} = \frac{N(\_ \cap \_)}{N(\_)}$$

$$= \boxed{\frac{239}{327} = 0.731}$$

**3.** (5 pts) If $P(A) = 0.6, P(B) = 0.5$, and $P(A \cup B) = 0.7$, are $A$ and $B$ independent events? Why or why not?

$P(A) \cdot P(B) = (0.6)(0.5) = 0.3$

Does $P(A \cap B) = P(A) \cdot P(B)$?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = 0.6 + 0.5 - P(A \cap B)$

$\Rightarrow P(A \cap B) = 0.4$

Since $P(A \cap B) \neq P(A) \cdot P(B)$

A + B are not independent events.

**4.** (5 pts) Die $A$ has orange on one face and blue on five faces. Die $B$ has orange on two faces and blue on four faces, and Die $C$ has orange on three faces and blue on three faces. All are fair dice. If the three dice are rolled, find the probability that exactly two of the three dice come up orange.

$P(\text{Exactly 2 Orange}) = P(E_1 \text{ or } E_2 \text{ or } E_3)$

$= P(E_1) + P(E_2) + P(E_3)$

$= \left(\frac{1}{6}\right)\left(\frac{2}{6}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{6}\right)\left(\frac{4}{6}\right)\left(\frac{1}{2}\right) + \left(\frac{5}{6}\right)\left(\frac{2}{6}\right)\left(\frac{1}{2}\right)$

$= \frac{1}{36} + \frac{2}{36} + \frac{5}{36} = \frac{8}{36} \approx 0.222$

| | A | B | C |
|---|---|---|---|
| $E_1$: | O | O | X |
| $E_2$: | O | X | O |
| $E_3$: | X | O | O |