Name: ______________________________ **Exam 2 - Main**

Math 421 Probability Theory – Crawford **14 November 2019**

Books and notes (in any form) are ***not*** allowed. You may use the given formula sheets and tables, and a calculator. But you must indicate any formulas used. Clearly indicate your answers and ***show all your work*** – partial credit may be given for written work. [There are 102 points available on the Main part of the exam. See the Exam 2 - P for more possible extra credit.] ***Good luck***!

Put all your work and answers on other paper and include this sheet as a cover sheet.

1. (10 pts) The telephone lines serving an airline reservation office are *all* busy about 60% of the time. If you and a friend are both calling this office, what is the probability that a total of four tries will be necessary for both of you to get through?

2. (12 pts) In the daily production of a certain kind of rope, the number of defects per foot Y is assumed to have a Poisson distribution with mean 2. The profit per foot when the rope is sold is given by X, where $X = 50 - 2Y - Y^2$. Find the expected profit per foot.

3. (14 pts) The length of time required by students to complete a one-hour exam is a random variable with a density function given by

$$f(y) = \begin{cases} cy^2 + y, & 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a). Find c.

(b). Find $F(y)$ and graph it.

(c). Given that a particular student needs at least 15 minutes to complete the exam, find the probability that she will require at least 30 minutes to finish.

4. (12 pts) The weekly amount of money spent on maintenance and repairs by a company was observed, over a long period of time, to be approximately normally distributed with mean \$500 and standard deviation \$25.

(a). If \$550 is budgeted for next week, what is the probability that the actual costs will exceed the budgeted amount?

(b). How much should be budgeted for weekly repairs and maintenance to provide that the probability the budgeted amount will be exceeded in a given week is only 0.1?

5. (12 pts) The strength Y of a certain material is such that its distribution is found by $Y = e^X$,where X is $N(10, 1)$.

(a). Find the cdf of Y.

(b). Compute $P(10,000 < Y < 20,000)$.

6. (12 pts) Let X be a random variable of the mixed type with cdf

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{2}, & 0 \leq x < 1 \\ \frac{1}{2}, & 1 \leq x < 2 \\ 1, & x \geq 3 \end{cases}$$

(a). Find $P(X = 2)$.

(b). Find $E[X]$.

7. (16 pts) Let the joint pmf of discrete random variables X and Y be defined by $f(x, y) = \dfrac{xy^2}{22}$ for $(1, 1), (1, 2), (1, 3), (2, 2)$.

(a). Find the marginal pmf $f_Y(y)$. [You may write $f_Y(y)$ in function form or express it in the margins of a table or dot graph.]

(b). Are X and Y independent? Justify your answer.

(c). Find $P(X = Y)$

(d). Find $\mathrm{Cov}(X, Y)$.

8. (14 pts) Given $f(x, y) = cxy,\ 0 \leq x \leq 1, x^2 \leq y \leq x$.

(a). Find the value of c for which $f(x, y)$ is a joint pdf for continuous random variables X and Y.

(b). Find μ_Y.