Exam 1 - Key

1\. (a) $P(\text{Match } 3W + 1P) = \dfrac{\binom{5}{3}\binom{64}{2}\binom{1}{1}}{\binom{69}{5}\binom{26}{1}}$

$= \dfrac{(10)(2016)(1)}{(11238513)(26)} \approx \boxed{6.899 \times 10^{-5}}$

.00006899

(b) Lose \$2 :
- Match 2 White and not the Powerball
- Match 1 " " " " "
- Match 0 " " " " "

$P(\text{Losing } \$2) = \dfrac{\binom{5}{2}\binom{64}{3}\binom{25}{1} + \binom{5}{1}\binom{64}{4}\binom{25}{1} + \binom{5}{0}\binom{64}{5}\binom{25}{1}}{\binom{69}{5}\binom{26}{1}}$

$= \dfrac{(10)(41664)(25) + (5)(635376)(25) + (1)(7624512)(25)}{(11238513)(26)}$

$\approx \boxed{0.9598}$

2\.

	Left-Thumb	Right-Thumb	
Left-eye	13	16	29
Right-eye	33	24	57
	46	40	86 ✓

(a) $P(LE \text{ and } LT) = \boxed{\frac{13}{86} \approx 0.1512}$

(b) $P(LE \text{ or } LT) = \frac{13+16+33}{86} = \boxed{\frac{62}{86} \approx 0.7209}$

(c) $P(LE \mid RT) = \boxed{\frac{16}{40} = 0.40}$

3. $P = (0.012)k$ probability of at least one accident during month w/ k days.

$1 - P = 1 - .012k$

Jan Feb Mar April

$P(1^{st}$ ~~Acc~~ Accident in April$) = (1-.012(31))(1-.012(28))(1-.012(31))(.012(30))$

$= (.628)(.664)(.628)(.36)$

$= \boxed{.09427}$

4. B_1 : 26% of sales ; prob of repair 0.10

B_2 : 34% " 0.05

B_3 : 31% " 0.04

B_4 : 9% " 0.04

$$P(B_2 \mid \text{Repair Needed}) = \frac{P(B_2 \cap \text{Repair Needed})}{P(\text{Repair Needed})}$$

$$= \frac{P(B_2)\,P(\text{Repair Needed} \mid B_2)}{\sum P(B_i)\,P(\text{Repaired Needed} \mid B_i)}$$

$$= \frac{(0.34)(.05)}{(.26)(.10) + (.34)(.05) + (.31)(.04) + (.09)(.04)}$$

$$= \frac{.017}{.059} \approx \boxed{0.2881}$$

5. $f(x) = \frac{3-|x-2|}{8}$, $X = 0, 1, 2, 3$

(a) $\mu = \sum x \cdot f(x) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{2}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{2}{8} = \frac{2+6+6}{8} = \frac{14}{8}$

$E(X) = \boxed{\frac{7}{4} = 1.75}$

$E(X^2) = \sum x^2 \cdot f(x) = 0^2 \cdot \frac{1}{8} + 1^2 \cdot \frac{2}{8} + 2^2 \cdot \frac{3}{8} + 3^2 \cdot \frac{2}{8} = \frac{2+12+18}{8} = \frac{32}{8}$
$= 4$

$\sigma^2 = Var(X) = \sum x^2 f(x) - \mu^2 = 4 - \left(\frac{7}{4}\right)^2 = \frac{64-49}{16} = \frac{15}{16} = \boxed{0.9375}$

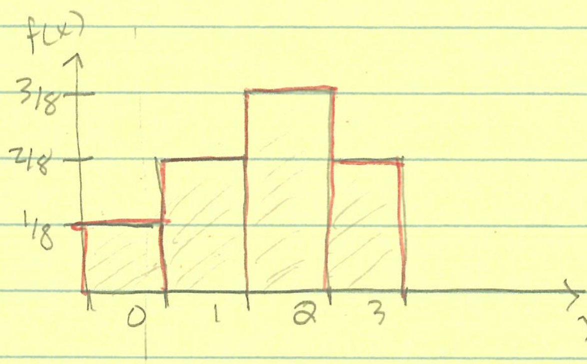
$E(X^3) = \sum x^3 \cdot f(x) = 0^3 \cdot \frac{1}{8} + 1^3 \cdot \frac{2}{8} + 2^3 \cdot \frac{3}{8} + 3^3 \cdot \frac{2}{8} = \frac{2+24+54}{8} = \frac{80}{8}$
$= 10$

$\gamma = \frac{E(X^3) - 3\mu\sigma^2 - \mu^3}{(\sigma^2)^{3/2}} = \frac{10 - 3\left(\frac{7}{4}\right)\left(\frac{15}{16}\right) - \left(\frac{7}{4}\right)^3}{\left(\frac{15}{16}\right)^{3/2}}$

$= \boxed{\cancel{-\frac{17}{8} \approx -2.125}}$

$\approx \boxed{-0.3098}$

(b)



6. 15% winning tickets. Purchase 12 $p = .15$, $n = 12$ binomial

(a) $P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1) = 1 - \text{binomcdf}(12, .15, 0)$

OR $1 - P(X=0) - P(X=1)$

$= 1 - \text{binompdf}(12, .15, 0) - \text{binompdf}(12, .15, 1)$ // same result.

$1 - 0.4435 \approx \boxed{0.5565}$

(b) $\mu = np = 12(.15) = \boxed{1.8}$

7. $M(t) = 0.3e^{t} + 0.5e^{2t} + 0.1e^{4t} + 0.1e^{6t}$

(a) $M'(t) = 0.3e^{t} + 1.0e^{2t} + 0.4e^{4t} + 0.6e^{6t}$

$E(X) = \mu = M'(0) = 0.3 + 1.0 + 0.4 + 0.6 = \boxed{2.3}$

$M''(t) = 0.3e^{t} + 2.0e^{2t} + 1.6e^{4t} + 3.6e^{6t}$

$M''(0) = 0.3 + 2.0 + 1.6 + 3.6 = 7.5$

$\sigma^2 = M''(0) - (M'(0))^2 = 7.5 - (2.3)^2 = \boxed{2.21}$

(b)

X	pmf $f(x)$
1	0.3
2	0.5
4	0.1
6	0.1

8.

Amount($) Loss X	Payment Y	Probability
0	0	0.900
500	0	0.068
1000	0	0.025
1500	500	0.005
2000	1000	0.002

\$1000 Deductible

$E(Y) = 0\cdot(0.900) + 0(.068) + 0(.025) + 500(.005) + 1000(.002)$

$= \boxed{4.5}$