1. <u>Constant Rule</u>: If $f(x) = c$, then $f'(x) = 0$. [Show $f'(a) = 0$ for arbitrary a.] $\frac{d}{dx}\left[c \right] = 0$

Proof:

2. If $f(x) = x$, then $f'(x) = 1$. [Show $f'(a) = 1$ for arbitrary a.] $\frac{d}{dx}\left[x \right] = 1$

Proof:

3. <u>Power Rule</u>: If $f(x) = x^n$ for any ***positive integer*** n , then $f'(x) = nx^{n-1}$.

[Show $f'(a) = na^{n-1}$ for arbitrary a.] $\frac{d}{dx}\left[nx^{n-1} \right]$

Proof:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \underline{\hspace{3cm}}$$

Factor the numerator:

$$= \lim_{x \to a} \frac{(x-a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \cdots + a^{n-2}x + a^{n-1})}{x - a}$$

You may want to multiply numerator to see that it really is $x^n - a^n$

$$= \lim_{x \to a} \underline{\hspace{10cm}}$$

$$= a^{n-1} + a \cdot a^{n-2} + a^2 \cdot a^{n-3} + \cdots + a^{n-2} \cdot a + a^{n-1}$$

$$= a^{n-1} + a^{n-1} + a^{n-1} + \cdots + a^{n-1} + a^{n-1}$$

$$= \underline{\hspace{3cm}}$$

Let $a \to x$ and therefore, $f'(x) = nx^{n-1}$ ■

4. <u>Constant Multiple Rule</u>: If f is differentiable at a point a, then cf is differentiable at a and

$(cf)'(a) = cf'(a)$ $$\frac{d}{dx}\Big[cf\Big] = c \cdot \frac{d}{dx}\Big[f\Big]$$

Proof: Let f be differentiable at a.

$$\begin{aligned}
(cf)'(a) &= \lim_{x \to a} \frac{(cf)(x) - (cf)(a)}{x - a} \\
&= \lim_{x \to a} \underline{\hspace{4cm}} \\
&= \lim_{x \to a} c \cdot \frac{f(x) - f(a)}{x - a} \\
&= \underline{\hspace{5cm}} \\
&= c \cdot f'(a) \text{ by definition of derivative. } \blacksquare
\end{aligned}$$

5. <u>Sum Rule</u>: If f and g are differentiable at a point a, then $f + g$ is differentiable at a and

$(f + g)'(a) = f'(a) + g'(a)$ $$\frac{d}{dx}\Big[f + g\Big] = \frac{d}{dx}\Big[f\Big] + \frac{d}{dx}\Big[g\Big]$$

Proof: Let f and g be differentiable at a.
$(f + g)'(a) =$

6. QUOTIENT RULE: If f and g are differentiable at a point a, then $\frac{f}{g}$ is differentiable at a (for $g(a) \neq 0$) and

$$\left(\frac{f}{g}\right)'(a) = \frac{g(a)f'(a) - f(a)g'(a)}{(g(a))^2} \qquad \frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$$

PROOF: Let f and g be differentiable at a and $g(a) \neq 0$.
Then g is continuous at a and $\exists$ and open interval containing a such that $g(x) \neq 0$.

$$\frac{(f/g)(x) - (f/g)(a)}{x-a} = \frac{\frac{f(x)}{g(x)} - \frac{f(a)}{g(a)}}{x-a} = \frac{\left[\frac{g(a)f(x) - f(a)g(x)}{g(x)g(a)}\right]}{(x-a)}$$

$$= \left[\frac{g(a)f(x) - f(a)g(x)}{g(x)g(a)(x-a)}\right] = \left[\frac{g(a)f(x) - f(a)g(x)}{(x-a)}\right] \cdot \underline{\qquad\qquad}$$

$$= \left[\frac{g(a)f(x) - g(a)f(a) + \underline{\qquad\qquad} - f(a)g(x)}{(x-a)}\right] \cdot \frac{1}{g(x)g(a)}$$

$$= \left[\frac{g(a)\left(\underline{\qquad\qquad}\right) - f(a)\cdot(g(x) - g(a))}{(x-a)}\right] \cdot \frac{1}{g(x)g(a)}$$

$$= \left[g(a)\cdot\frac{f(x) - f(a)}{x-a} - f(a)\cdot \underline{\qquad\qquad}\right] \cdot \frac{1}{g(x)g(a)}$$

Then

$$\lim_{x\to a} \frac{(f/g)(x) - (f/g)(a)}{x-a} = \lim_{x\to a}\left[g(a)\cdot\frac{f(x) - f(a)}{x-a} - f(a)\cdot\frac{g(x) - g(a)}{x-a}\right] \cdot \frac{1}{g(x)g(a)}$$

$$= \left[\lim_{x\to a} g(a) \cdot \underline{\qquad\qquad} - \lim_{x\to a} f(a) \cdot \lim_{x\to a} \frac{g(x) - g(a)}{x-a}\right] \cdot \lim_{x\to a} \frac{1}{g(x)g(a)}$$

$$= \left[g(a)\cdot f'(a) - \underline{\qquad\qquad}\right] \cdot \frac{1}{g(a)g(a)}$$

i.e $\underline{\qquad\qquad} = \dfrac{g(a)\cdot f'(a) - f(a)\cdot g'(a)}{(g(a))^2}$ by definition of derivative. ■

7. <u>PRODUCT RULE</u>: If f and g are differentiable at a point a, then fg is differentiable at a and

$(fg)'(a) = f(a)g'(a) + g(a)f'(a)$ $\qquad \frac{d}{dx}\left[fg\right] = fg' + gf'$

PROOF: Let f and g be differentiable at a

$(fg)'(a) =$

8. Prove: If $f(x) = x^n$ for negative integer n, then $f'(x) = nx^{n-1}$.
[Hint: Let $m = -n > 0$ and write as a fraction and use the previous theorems/rules.]
PROOF:

Note: Although we've only proven the power rule for positive and negative *integers*, it holds for any real number. (We may prove later, if given time)

Homework: Finish Worksheet(s) and Section 28: #1, 2, 3, 6, 7, 14(a)