Geometric Series is of the form $\sum_{n=m}^{\infty} ar^n, \ a \neq 0$

1. Consider $m = 0$. For which values of r, does the series $\sum_{n=0}^{\infty} ar^n, \ a \neq 0$ converge.

Case 1 r = 1: $\sum_{n=0}^{\infty} a(1)^n = \underline{\sum_{n=0}^{\infty} a}$

Then the k-th partial sum is

$$
\begin{aligned}
s_k &= \sum_{n=0}^{k} a = a + a + a + \cdots + a = \underline{(k+1)a} \\
\text{i.e. } s_k &= \underline{(k+1)a} \\
\text{Then } \lim_{k\to\infty} s_k &= \begin{cases} \underline{+\infty}, & \text{if } a > 0 \\ \underline{-\infty}, & \text{if } a < 0 \end{cases}
\end{aligned}
$$

Thus, the sequence of partial sums diverges.

Therefore, the infinite series $\sum_{n=0}^{\infty} a$ $\underline{\text{diverges}}$.

Case 2 r = -1: $\sum_{n=0}^{\infty} a(-1)^n$

Then the k-th partial sum is

$$
\begin{aligned}
s_k &= \sum_{n=0}^{k} a(-1)^n = a - a + a - a + \cdots \pm a = \begin{cases} \underline{0}, & \text{if } k \text{ is odd} \\ \underline{a}, & \text{if } k \text{ is even} \end{cases} \\
\text{i.e. } s_k &= (a, 0, a, 0, \ldots) \\
\Rightarrow \lim_{k\to\infty} s_k & \quad \underline{DNE}.
\end{aligned}
$$

Therefore the infinite series $\sum_{n=0}^{\infty} a(-1)^n$ diverges.

Case 3 $r \neq 1$:

2. Homework: For $|r| < 1$, prove the general formula that $\displaystyle\sum_{n=m}^{\infty} ar^n = \frac{ar^m}{1-r}$

3. Homework: Determine whether the following geometric series converge or diverge. If they converge, find the value of the sum.

(a). $\displaystyle\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$ **(b).** $\displaystyle\sum_{n=1}^{\infty} \frac{-2\cdot 3^n}{2^n}$ **(c).** $\displaystyle\sum_{n=2}^{\infty} \frac{3^n}{4^{n+1}}$ **(d).** $\displaystyle\sum_{n=0}^{\infty} \frac{e^n}{2^{n-1}}$

(e). $\displaystyle\sum_{n=1}^{\infty} \frac{(-6)^{n-1}}{5^{n-1}}$ **(f).** $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{4^{-n+1}}$ **(g).** $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{3^{2n+1}}$ **(h).** $3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \frac{3}{16} - \cdots$