<u>Ex</u> $(s_n) = (-1)^n$

(a). Does the $\lim s_n$ exist?

(b). How many terms get close to (or are) 1?

(c). How many terms get close to (or are) -1?

So even though $\lim s_n$ DNE, ________ behaves like an upper *limit* and ________ behaves like a lower *limit*.

<u>Ex</u> $s_n = \begin{cases} 2 - \dfrac{1}{n}, & \text{if } 3 \nmid n \\ \dfrac{1}{n}, & \text{if } 3 | n \end{cases}$

(a). Sketch the sequence.

(b). Does s_n converge?

(c). How many terms approach 2?

(d). How many terms approach 0?

So even though $\lim s_n$ DNE, ________ behaves like an upper *limit* and ________ behaves like a lower *limit*.

<u>Ex</u> $s_n = 2 - \dfrac{1}{n}$

(a). Does s_n converge?

(b). How many terms approach 2?

(c). What is the lower *bound* for s_n?
How many terms approach this lower *bound*?

So 1 is a lower *bound* for s_n, but it is not a lower *limit* for s_n because infinitely many values do not approach it as n gets larger.

<u>**Def**</u> Let s_n be a sequence in $\mathbb{R}$. Then

$$\limsup s_n = \lim_{N\to\infty} \sup\{s_n \mid n > N\}$$

$$\liminf s_n = \lim_{N\to\infty} \inf\{s_n \mid n > N\}$$

Huh?!?

- Create a sequence of sets $S_N = \{s_n \mid n > N\}$
- Find the supremum of each set S_N and create a new sequence of these suprema. i.e., $(\sup S_N)$
- Find the limit of this new sequence. i.e., Find the limit of the sequence of the suprema. i.e., Find $\lim\limits_{N\to\infty} \sup S_N$.

1. $s_n = (-1)^n$

2. $s_n = \begin{cases} 2 - \dfrac{1}{n}, & \text{if } 3 \nmid n \\ \dfrac{1}{n}, & \text{if } 3 | n \end{cases}$

$N = 1 : S_1 = \{s_n \mid n > 1\} = \left\{\frac{3}{2}, \frac{1}{3}, \frac{7}{4}, \frac{9}{5}, \frac{1}{6}, \frac{13}{7} \dots \right\} \Longrightarrow \inf S_1 =$ $\qquad$ $\sup S_1 =$

$N = 2 : S_2 =$

$N = 3 : S_3 =$

Repeat the process until you feel confident answering:

$\liminf s_n = \lim\limits_{N\to\infty} \inf S_N =$ ________ and $\limsup s_n = \lim\limits_{N\to\infty} \sup S_N =$ ________

Note:

$\limsup s_n$ is the ____________ value that ______________ many terms of s_n get close to.

$\liminf s_n$ is the _____________ value that ______________ many terms of s_n get close to.

3. $s_n = \begin{cases} 5, & n < 100 \\ 2 - \dfrac{1}{n}, & n \geq 100 \end{cases}$

$S_1 = \{s_n \mid n > 1\} = \left\{5, 5, 5, 5, \ldots, 5, \frac{199}{100}, \frac{201}{101} \frac{203}{102}, \frac{205}{103}, \frac{207}{104} \ldots\right\} \Longrightarrow \inf S_1 =$ $\qquad \sup S_1 =$

$S_2 =$

$\vdots$

$S_{98} =$

$S_{99} =$

$S_{100} =$

$S_{101} =$

Repeat the process until you feel confident answering:

$\liminf s_n = \lim_{N \to \infty} \inf S_N =$ ________ and $\limsup s_n = \lim_{N \to \infty} \sup S_N =$ ________

Compare $\limsup s_n$ with $\sup \{s_n \mid n \in \mathbb{N}\}$ Are they the same or different?