<u>Def</u> The set of ______________________ is the set $\{1, 2, 3, \ldots\}$ and denoted ________ .

<u>Def</u> The **<u>successor</u>** of a natural number is ______________________ .

i.e. For each $n \in \mathbb{N}$, the successor is ___________ .

Similar definition for predecessor.

<u>Peano Axioms</u> (___________________)

N1. $1 \in \mathbb{N}$

N2. If $n \in \mathbb{N}$, then $n + 1 \in \mathbb{N}$

N3. 1 is not a successor of any $n \in \mathbb{N}$

N4. If $n, m \in \mathbb{N}$ have the same successor, then $n = m$.

N5. If $S \subseteq \mathbb{N}$ and $1 \in S$ and $\forall n \in S, n + 1 \in S$, then $S = \mathbb{N}$

<u>Mathematical Induction is a Direct Consequence of N5</u>

$$S \subseteq \mathbb{N}$$

$$1 \in \mathbb{N}$$

$$\text{If } n \in S, \text{then } n + 1 \in S$$

$$S = \mathbb{N}$$

<u>Def</u> The set of **<u>Natural Numbers</u>** is $\{1, 2, 3, \ldots\}$ and denoted $\mathbb{N}$.

If $n, m \in \mathbb{N}$,

1. Is $n + m \in \mathbb{N}$?

2. Is $n - m \in \mathbb{N}$?

<u>Def</u> The set of **<u>Integers</u>** is ______________________________ and denoted $\mathbb{Z}$.

If $n, m \in \mathbb{Z}$,

3. Is $n \cdot m \in \mathbb{Z}$?

4. Is $\frac{m}{n} \in \mathbb{Z}$?

<u>Def</u> The set of **<u>Rational Numbers</u>** is the set of all numbers of the form __________________________________
The set is denoted $\mathbb{Q}$.

Notes:

- Avoid duplicate numbers in $\mathbb{Q}$ by considering
- Are terminating decimals in $\mathbb{Q}$? e.g. $3.741 =$
- Are repeating decimals in $\mathbb{Q}$? e.g. $0.33\overline{3} =$
- Are all decimals in $\mathbb{Q}$?

<u>DEF</u> An <u>**Algebraic Number**</u> is a number that is

i.e. An algebraic number is any number $x = r$ that satisfies an equation of the form

$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 = 0$ where $a_0, a_1, \ldots, a_n \in \mathbb{Z}$, $a_n \neq 0$ and $n \geq 1$.

Are all algebraic numbers not rational?

Are all rational numbers $x = \dfrac{m}{n}$ algebraic?

Are all numbers that are not rational also algebraic?