

Name: _____

Take-Home Quiz 2

Math 381 Advanced Calculus – Crawford

Due: Friday, November 8, 2019 by 12:45 pm

This is a take-home quiz. **You are on your honor to work alone. You may not get help from other people (in person or electronically).** Put all of your work and answers on your own paper. Attach this sheet as a cover sheet. You must *show all your work* for full credit. Good Luck! [Scores will be scaled to 20 points.]

1. (8 pts) Given $s_n = n^2 \sin\left(\frac{n\pi}{2}\right)$,

- (a). Give an example of a *nonconstant* monotone subsequence. If one does not exist, clearly state so.
- (b). Give the set of subsequential limits.
- (c). Does the sequence s_n converge, diverge to $+\infty$, diverge to $-\infty$, or none of these options?

2. (8 pts) Determine whether the following statements are TRUE or FALSE. If it is false clearly explain the reason why or give a counterexample.

(a). $\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{6}, \frac{1}{12}, \frac{1}{10}, \dots\right)$ is subsequence of $(s_n) = \left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots\right)$.

- (b). If s_n is unbounded above, then s_n contains a subsequence that has $+\infty$ as a limit
- (c). Every oscillating sequence has a convergent subsequence.

3. (8 pts) Prove the general formula for the geometric series $\sum_{n=m}^{\infty} ar^n = \frac{ar^m}{1-r}$, for $|r| < 1$.

4. (8 pts) Determine whether the following geometric series converge or diverge. If it converges, find the sum.

(a). $\sum_{n=1}^{\infty} \frac{3^{-2n+1}}{(-2)^{n-1}}$

(b). $\sum_{n=2}^{\infty} \frac{2^n}{3^{1-n}}$

5. (8 pts) Determine whether or not the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}}$ converges. Show work and justify your answer.

[Hint: Rationalize and look at the partial sums.]