1. $s_n = n^2 \sin\left(\frac{n\pi}{2}\right)$

$(1, 0, -9, 0, 25, 0, -49, \ldots)$

(a) $n_k = 3+4k \quad k = 0, 1, 2, \ldots$

or $n_k = 4k-1, \; k = 1, 2, 3 \ldots \Rightarrow s_{4k-1} = (-9, -49, -169, \ldots)$

OR $n_k = 4k-3 \Rightarrow s_{4k-3} = (1, 25, 81, \ldots)$

(b) $S = \{0, +\infty, -\infty\}$

(c) It does not converge to $s \in \mathbb{R}$, nor diverge to $+\infty$, nor diverges to $-\infty$.

2. (a) False, the order of the original sequence must be maintained in a subsequence.

(b) True

(c) False. Counterexample: $s_n = (-1)^n n^2$
$= (-1, 4, -9, 16, \ldots)$

3. Let $|r|<1$. [Show $\sum_{n=m}^{\infty} ar^n$ converges to $\frac{ar^m}{1-r}$

Consider $s_k = \sum_{n=m}^{k} ar^n = ar^m + ar^{m+1} + \ldots + ar^k$

then $rs_k = \qquad ar^{m+1} + \ldots + ar^k + ar^{k+1}$

$\Rightarrow s_k - rs_k = ar^m + ar^{m+1} + \ldots + ar^k$
$\qquad - ar^{m+1} - \ldots - ar^k - ar^{k+1}$

$s_k(1-r) = ar^m - ar^{k+1}$

$\Rightarrow s_k = \dfrac{ar^m - ar^{k+1}}{1-r}$

$\lim_{k\to\infty} s_k = \lim_{k\to\infty}\left(\frac{ar^m}{1-r} - \frac{ar^{k+1}}{1-r}\right) = \frac{ar^m}{1-r} - 0$

since $|r|<1$

Since $\lim_{k\to\infty} s_k = \frac{ar^m}{1-r}$, then by definition of a converging series, $\sum_{n=m}^{\infty} ar^n$ converges to $\frac{ar^m}{1-r}$

4. (a) $\sum_{n=1}^{\infty} \frac{3^{-2n+1}}{(-2)^{n-1}} = \sum \frac{3^{-2n} 3^1}{(-2)^n (-2)^{-1}} = \sum \frac{(3^2)^{-n} 3 \cdot (-2)}{(-2)^n}$

$= \sum \frac{-6}{9^n (-2)^n} = \sum -6 \cdot \left(\frac{1}{-18}\right)^n$

$|r| = \frac{1}{18} < 1$

$\therefore$ converges to $\frac{-6(-\frac{1}{18})}{1-(-\frac{1}{18})} = \frac{\frac{6}{18}}{\frac{19}{18}} = \frac{6}{18} \cdot \frac{18}{19}$

$= \boxed{\frac{6}{19}}$

(b) $\sum_{n=2}^{\infty} \frac{2^n}{3^{1-n}} = \sum \frac{2^n}{3^1 \cdot 3^{-n}} = \sum \frac{2^n \cdot 3^n}{3} = \sum \frac{1}{3}(6)^n$

$\Rightarrow |r| = 6 > 1$ $\boxed{\text{Diverges}}$ by GST.

5. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}} \cdot \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}-\sqrt{n}} = \sum \frac{\sqrt{n+1}-\sqrt{n}}{n+1-n}$

$= \sum (\sqrt{n+1} - \sqrt{n})$

Consider

$S_K = (\cancel{\sqrt{2}} - \sqrt{1}) + (\cancel{\sqrt{3}} - \cancel{\sqrt{2}}) + (\cancel{\sqrt{4}} - \cancel{\sqrt{3}}) + \dots + (\cancel{\sqrt{K}} - \cancel{\sqrt{K-1}}) + (\sqrt{K+1} - \cancel{\sqrt{K}})$

So $S_K = -\sqrt{1} + \sqrt{K+1}$

$\lim_{K \to \infty} S_K = \lim_{K \to \infty} -\sqrt{1} + \sqrt{K+1} = +\infty$

Since the sequence of partial sums diverges, then the series $\sum \frac{1}{\sqrt{n+1}+\sqrt{2}}$ diverges.