Name: _____ **Quiz 1**

Math 381 Advanced Calculus – Crawford **20 September 2019**

Books, notes (in any form), and calculators are not allowed. You may use a sheet of Field Properties and their Consequences. *Show all your work*. Good Luck!

1. (6 pts) Using only the Field Properties and Consequences of Field Properties, prove the following.
[Clearly justify each step by indicating which properties you use.]

Let $a \in F$, where F is a field. If $a \neq 0$, then

(a). $a^{-1} \neq 0$ and

(b). $(a^{-1})^{-1} = a$.

2. (4 pts) Use induction to prove the following:

If $0 < x < y$, then $x^n < y^n$ for all $n \in \mathbb{N}$.

3. (4 pts) For the following subsets of $\mathbb{R}$, state the minimum, maximum, infimum, and supremum, if they exist. If it does not exist, clearly state this.

(a). $S = \left\{ \dfrac{n}{n+1} \;\middle|\; n \in \mathbb{N} \right\}$

(b). $T = \left\{ r \in \mathbb{Q} \;\middle|\; r^2 < 5 \right\}$

4. (6 pts) Determine whether the following statements are TRUE or FALSE.
If it is FALSE, give a counterexample. If it is TRUE, no additional work needed.

T F $\forall a, b \in \mathbb{R}$, if $a < b$, then $|a| < |b|$.

T F $\forall a, b \in \mathbb{R}$, $|a - b| \leq |a| + |b|$.

T F Let S be a nonempty bounded subset of $\mathbb{R}$. If $\sup S \in S$, then $\sup S = \max S$.