Name: __________________________________ **Exam 1**

Math 381 Advanced Calculus – Crawford **02 October 2019**

Score	
1	/16
2	/16
3	/16
4	/16
5	/16
6	/16
7	/16
Total	/100

- Books and notes (in any form) are not allowed.
- You may use the given sheet with the Field Properties and their Consequences.
- There are 112 points possible on the exam. Any points you receive over 100 will be cut in thirds, so that the highest possible grade is 104 out of 100.
- Do all of your work on separate paper and attach this sheet as a cover sheet.

Show all your work. Good Luck!

1. (16 pts). Give precise and accurate definitions/statements of the following.

(a). A form of the Triangle Inequality other than $|a+b| \leq |a| + |b|$.

(b). A set S bounded below

(c). The Archimedean Property

2. (16 pts). For each of the following sets, state the minimum, maximum, infimum and supremum, if they exist. If it does not exist, clearly state this.

(a). $A = \bigcap_{n=1}^{\infty} \left(2 + \frac{1}{n}, 5 + \frac{1}{n}\right]$ **(b).** $B = \left\{ \frac{4-x}{2x} \;\middle|\; x \geq 1, x \in \mathbb{R} \right\}$

3. (16 pts). Given the recursively defined sequence $s_1 = 3$ and $s_{n+1} = \frac{1}{4}(s_n + 1)$

(a). Determine the first four terms of the sequence, i.e., s_1, s_2, s_3, s_4.

(b). Assume that (s_n) converges and find the limit.

4. (16 pts). Use the $\epsilon - N$ definition of convergence to prove that $\lim \frac{1-2n^2}{3n+4n^2} = -\frac{1}{2}$.

5. (16 pts). If s_n converges to s and t_n converges to t, prove that $\lim(s_n + t_n) = s + t$.

6. (16 pts). Let a, b, c be elements of an ordered field F. Prove (vii) of the CoOFP:

If $0 < a < b$, then $0 < b^{-1} < a^{-1}$ [You may use any of the other Field and Ordered Field Properties and Their Consequences.]

7. (16 pts). New Proof: Let S be a nonempty bounded subset of $\mathbb{R}$ and let $k \in \mathbb{R}$. Define $T = \{ks \mid s \in S\}$. Prove that if $k < 0$, then $\sup T = k \cdot \inf S$.