Let A be an $n \times n$ square matrix. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be the linear transformation defined as $T(\mathbf{x}) = A\mathbf{x}$.
Then the following statements are equivalent (i.e. they are either all true or all false).

Fill in the blanks so that the statements are equivalent. Indicate which previous statement(s) and theorem/definition (section and theorem/definition number) imply each statement.
[Hint: One or more of the previous statements in the IMT will likely be the "if" part of the appropriate theorem to use.]

1. A is an invertible matrix. [To complete the circle, explain why #13 ⇒ #1.]
By (13) and Theorem 4 (Sec. 3.2).

2. There is an $n \times n$ matrix D such that $AD =$ ___I_n___.
By (1), A^{-1} exists and let $D = A^{-1}$ in the definition of inverse (Sec. 2.2).

3. There is an $n \times n$ matrix C such that $CA =$ ___I_n___.
By (1), A^{-1} exists and let $C = A^{-1}$ in the definition of inverse (Sec. 2.2).

4. The equation $A\mathbf{x} = \mathbf{0}$ has only ___the trivial___ solution.
By (1) and Theorem 5 (Sec. 2.2) the equation $A\mathbf{x} = \mathbf{0}$ has the unique solution $\mathbf{x} = A^{-1}\mathbf{0} = \mathbf{0}$.

5. The columns of A are ___linearly independent___ .
By (4) and Statement (i.e. Theorem) (Sec. 1.7).

6. For each $\mathbf{b}$ in $\mathbb{R}^n$, the equation $A\mathbf{x} = \mathbf{b}$ has ___a unique___ solution.
By (1) and Theorem 5 (Sec. 2.2).

7. The columns of A span ___$\mathbb{R}^n$___ .
By (6) and Theorem 4(c) (Sec. 1.4).

8. A has ___n___ pivot positions.
By (7) and Theorem 4(d) (Sec. 1.4).

9. A is row equivalent to ___I_n___
By (1) and Theorem 7(d) (Sec. 2.2).

10. A^T is an ___invertible___ matrix.
By (1) and Theorem 6(c) (Sec. 2.2).

11. T maps $\mathbb{R}^n$ ___onto___ $\mathbb{R}^n$.
By (7) and Theorem 12(a) (Sec. 1.9).

12. T is a ___one-to-one___ transformation.
By (5) and Theorem 12(b) (Sec. 1.9).

13. $\det A \neq 0$.
By (1) and Theorem 4 (Sec. 3.2).