Interchange, Scaling, and Replacement are called ______________________________ for matrices.

<u>DEF</u> Two matrices are ____________________ if there is a sequence of elementary row operations that

<u>DEF</u> A __________________ of a row is the left-most nonzero entry in that row.

<u>EX</u>: (from previous worksheet) $\begin{bmatrix} 1 & 2 & 3 & 14 \\ 0 & -4 & 5 & 33 \\ 2 & -1 & 1 & 13 \end{bmatrix}$

We found a row equivalent matrix of the one above that was in a "good" form (step 4 of previous worksheet):

$$\begin{bmatrix} 1 & 2 & 3 & 14 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 9 & 45 \end{bmatrix}$$

But we went further to find a row equivalent matrix in an even "better" form (step 7):

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

<u>EX</u>: $\begin{bmatrix} -8 & -4 & -6 & -2 & 4 \\ 0 & 0 & 3 & 6 & 3 \\ 4 & 2 & 1 & 0 & -4 \\ 0 & 0 & 2 & 1 & 2 \end{bmatrix} \Longrightarrow \begin{bmatrix} 4 & 2 & 3 & 1 & -2 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 & -\frac{5}{4} \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

<u>DEF</u> A rectangular matrix is in ______________________________ if it has the following 3 properties:

1.

2.

3.

<u>DEF</u> Furthermore, it is in ______________________________________ if these 2 additional properties hold:

4.

5.

THEOREM The Reduced (Row) Echelon Form is ____________ for any given matrix.

DEF A __________________ in a matrix is the _____________ of a leading 1 in reduced echelon form.

DEF A __________________ is a column that contains a __________________ .

DEF A ___________ is a nonzero number in the pivot position used to

Row Reduction Algorithm
(variant of Gaussian Elimination)

FORWARD PHASE
(to echelon form)

Step 1
Locate the _____________ nonzero column and note:

- This is a __________________
- The __________________ is at the top of this column

Step 2
Choose a nonzero number in this column to be the ___________ .

- Choose wisely
- If necessary, interchange rows to move it to the pivot position
- (optional) Scale row to get a 1 in the pivot position.

Ex:

$$\begin{array}{rcrcrcrcr} & & -5x_2 & + & x_3 & + & x_4 & = & 5 \\ 2x_1 & + & x_2 & + & 3x_3 & + & 3x_4 & = & 11 \\ x_1 & - & 2x_2 & + & 2x_3 & + & 2x_4 & = & 8 \\ 5x_1 & & & & & + & 2x_4 & = & 20 \end{array}$$

$$\begin{bmatrix} 0 & -5 & 1 & 1 & 5 \\ 2 & 1 & 3 & 3 & 11 \\ 1 & -2 & 2 & 2 & 8 \\ 5 & 0 & 0 & 2 & 20 \end{bmatrix}$$

Step 3
Use ____________________ to get all zero entries below the pivot

Step 4
Ignore/Cover all rows above and including the pivot position.
____________________________________ until echelon form attained.

BACKWARD PHASE
(to ***reduced*** echelon form)

Step 5
Locate the rightmost __________ .

(a). Scale row to make ______________ .

(b). Use Row Operations to get ________________ above the pivot.

(c). Locate the next rightmost pivot. Repeat steps 5(a)-5(b) until reduced echelon from is attained.

$$\begin{bmatrix} 1 & -2 & 2 & 2 & 8 \\ 0 & 5 & -1 & -1 & -5 \\ 0 & 0 & -8 & -6 & -10 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

[Extra space for previous problem, if needed.]

$$\text{Linear System} \Rightarrow \text{Matrix} \Rightarrow \text{REF} \Rightarrow$$

$$\begin{array}{rcrcrcrcr} & & -5x_2 & + & x_3 & + & x_4 & = & 5 \\ 2x_1 & + & x_2 & + & 3x_3 & + & 3x_4 & = & 11 \\ x_1 & - & 2x_2 & + & 2x_3 & + & 2x_4 & = & 8 \\ 5x_1 & & & & & + & 2x_4 & = & 20 \end{array} \Rightarrow \begin{bmatrix} 0 & -5 & 1 & 1 & 5 \\ 2 & 1 & 3 & 3 & 11 \\ 1 & -2 & 2 & 2 & 8 \\ 5 & 0 & 0 & 2 & 20 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 2 & 2 & 8 \\ 0 & 5 & -1 & -1 & -5 \\ 0 & 0 & -8 & -6 & -10 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

$$\text{RREF} \Rightarrow \text{System} \Rightarrow \text{Solution}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2/5 & 4 \\ 0 & 1 & 0 & -1/20 & -3/4 \\ 0 & 0 & 1 & 3/4 & 5/4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{ccccrcr} x_1 & & & + & 2/5x_4 & = & 4 \\ & x_2 & & - & 1/20x_4 & = & -3/4 \\ & & x_3 & + & 3/4x_4 & = & 5/4 \\ & & & & 0 & = & 0 \end{array} \Rightarrow \left\{ \begin{array}{ccrcr} x_1 & = & 4 & - & 2/5x_4 \\ x_2 & = & -3/4 & + & 1/20x_4 \\ x_3 & = & 5/4 & - & 3/4x_4 \\ x_4 & is & free & & \end{array} \right.$$

<u>Def</u> The ____________________ are the variables corresponding to the ________________ .

<u>Ex</u>:

<u>Def</u> Any remaining variables not associated with the pivot columns are called ________________ .

<u>Ex</u>:

$\Longrightarrow$ Solution: