Ex: Suppose a population of fruit flies grows at a rate proportional to the number of fruit flies present. It is found that the growth rate is 32% each day.

(a). If you start with $p_0 = 10$ fruit flies, create a sequence that gives the (approximate) population of fruit flies each day. If you go on vacation for 10 days and forget to throw out your fruit, how many fruit flies will you have when you get back?

Day	Population		
0	$p_0 =$		10
1	$p_1 =$	$10 + .32(10) =$	13.2
2	$p_2 =$	$13.2{+}.32(13.2) =$	17.424
3	$p_3 =$	$17.424{+}.32(17.424) =$	22.9997
4	$p_4 =$		30.3596
5	$p_5 =$		40.0746
6	$p_6 =$		52.8985
7	$p_7 =$		69.8261
8	$p_8 =$		92.1704
9	$p_9 =$		121.6649
10	$p_{10} =$		160.5977

(b). Find a general formula for the population after k days.

Dynamical System: Equation(s) that describe the relationships between quantities that change in time .

- Discrete Dynamical System : Study the behavior over discrete time intervals.

  $\Rightarrow$ Difference Equations

  Ex:

- Continuous Dynamical System: Study behavior over continuous time intervals.

  $\Rightarrow$ Differential Equations

  Ex:

Often interested in long-term behavior. That is, as $t \to \infty$ (or equivalently $k \to \infty$), do the solutions

- Grow or decrease without bound?
- Approach 0?
- Approach a finite, nonzero value?
- Oscillate?
- Exhibit chaotic behavior?

Fruit fly example was a single difference equation of the form:

$x_{k+1} = ax_k$, $k = 0, 1, 2, \ldots$ and x_0 is given. The solution is the sequence of values $x_0, x_1, \ldots$.

Extend this form for a system (more than one equation/relationship) of difference equations:

$\mathbf{x}_{k+1} = A\mathbf{x}_k$ for $k = 0, 1, 2, \ldots$ and $\mathbf{x}_0$ is the initial vector.

The sequence of vectors $\{\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots\}$ describes the state of the system at time k.

Still interested in long-term behavior. That is, as $t \to \infty$ (or equivalently $k \to \infty$), does the solution vector $\mathbf{x}_k$

- Grow or decrease without bound?
- Approach the zero vector $\mathbf{0}$?
- Approach a finite, nonzero vector $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$?
- Oscillate?
- Exhibit chaotic behavior?

Let's look at some examples first before answering some of these questions (and seeing their connection to eigenvalues).

Ex: A car rental company has 800 cars and 3 locations: O'Hare, Midway, and Downtown. Cars may be returned to any location regardless of where they were rented. Each week they find that

- 90% of cars rented at O'Hare are returned to O'Hare, 1% returned to Midway, and 9% returned downtown.
- 92% of cars rented at Midway are returned to Midway, 4% returned downtown, and 4% returned to O'Hare.
- 83% of cars rented downtown are returned to downtown, 12% returned to O'Hare, and 5% returned to Midway

Initially O'Hare has 400 cars, Midway has 250 cars, and downtown has 150 cars.

(a). Set up the dynamical system as a matrix difference equation.

(b). Find how many cars are at each location after 1 month.

(c). Does it seem that the number of cars in each location is approaching a fixed value?

Ex: Demographic studies for a particular city show that each year 6% of the city population moves to the suburbs while 94% stays in the city and 2% of the suburb population moves to the city while 98% stays in the suburbs. In 2012, the city population is $835,000$ and the suburbs is $360,000$. [Ignore other factors on population change such as death, birth, migration into and out of the region.]

(a). Set up the dynamical system as a matrix difference equation to describe the population change.

(b). Find the population in the city and the suburbs in 2015 (i.e. Find ___$\mathbf{x}_3$___).

Section 1.10, p. 87: #9, 10, 11, 12[Also, find how many cars are in each location after 1 week.]