Name: Key

Math 362 Linear Algebra – Crawford

Quiz 3

24 October 2018

Books, notes (in any form), and calculators are not are allowed. Show all other work for credit. ***Good luck!*** [Note: Each quiz score will be scaled to 15 points after grading.]

1. (7 pts) Given the system $\begin{array}{rcrcl} 2sx_1 & + & 3x_2 & = & 4 \\ 6x_1 & + & sx_2 & = & 2 \end{array}$, which contains the parameter s.

(a). Determine the value(s) of s for which the system has a unique solution.

$$\begin{bmatrix} 2s & 3 \\ 6 & s \end{bmatrix} \vec{x} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\det(A) = 2s^2 - 18 = 0$$

$$s^2 = 9$$

$$s = \pm 3$$

$\boxed{\text{Unique Sol}^{\text{n}} \text{ for all } s, \text{ except } s \neq \pm 3}$

(b). Use Cramer's Rule to find the solution.

$$x_1 = \frac{\begin{vmatrix} 4 & 3 \\ 2 & s \end{vmatrix}}{\det A} = \boxed{\frac{4s-6}{2s^2-18}} = \frac{2s-3}{s^2-9}$$

$$x_2 = \frac{\begin{vmatrix} 2s & 4 \\ 6 & 2 \end{vmatrix}}{\det A} = \boxed{\frac{4s-24}{2s^2-18}} = \frac{2s-12}{s^2-9} = \frac{2(s-6)}{s^2-9}$$

2. (4 pts) Determine whether the following statements are true or false. Note: Explanations not required

(a). If A is invertible, then the columns of A^{-1} are linearly independent. $\boxed{\text{TRUE}}$

Then A^{-1} exists and is also invertible

$\therefore$ col^{n}s of A^{-1} lin indep by IMT

(b). If $n \times n$ matrices satisfy the property that $EF = I$, then E and F commute. $\boxed{\text{TRUE}}$

Then E & F are invertible by IMT

$\Rightarrow E^{-1} = F$ by def of Inverse. $\Rightarrow EF = EE^{-1} = E^{-1}E = FE = I$

(c). If A is an $n \times n$ matrix such that $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$, then the solution is unique for each $\mathbf{b}$.

ie The col^{n}s of A span $\mathbb{R}^n$ $\boxed{\text{TRUE}}$

$\Rightarrow A$ is invertible by IMT

$\Rightarrow A\vec{x} = \vec{b}$ has a unique sol^n by IMT

(d). Suppose A is a $n \times n$ matrix with $\det A = 1$. If the entries in A are integers, then the entries in A^{-1} are integers.

$\boxed{\text{TRUE}}$ $A^{-1} = \frac{1}{\det A}\,\mathrm{adj}(A) = \mathrm{adj}(A)$ and if the entries of A are integers, so will the entries of adj(A)

3. (4 pts) Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. If $T(\mathbf{u}) = T(\mathbf{v})$ for a pair of distinct vectors $\mathbf{u}$ and $\mathbf{v}$, prove that T is not onto $\mathbb{R}^n$.

Proof Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation

Let $T(\vec{u}) = T(\vec{v})$ for a pair of distinct vectors $\vec{u}$ & $\vec{v}$.

Then T is not one-to-one by def.

Let A be the standard matrix s.t. $T(\vec{x}) = A\vec{x}$

Then A is not invertible by the IMT.

$\therefore$ T is not onto $\mathbb{R}^n$ by the IMT. ∎

Alternate:

Proof: Let $T: \mathbb{R}^n \to \mathbb{R}$ be a linear transformation.

Let $T(\vec{u}) = T(\vec{v})$ for a pair of distinct vectors $\vec{u}$ & $\vec{v}$.

$\Rightarrow T(\vec{u}) - T(\vec{v}) = \vec{0} \Rightarrow T(\vec{u} - \vec{v}) = \vec{0}$

ie $\vec{u} - \vec{v}$ is a sol^n to $T(\vec{x}) = \vec{0}$

But $\vec{u} - \vec{v} \neq \vec{0}$ since $\vec{u}$ & $\vec{v}$ are distinct

ie There exists a nontrivial sol^n to $T(\vec{x}) = \vec{0}$

$\Rightarrow A\vec{x} = \vec{0}$ has a nontrivial sol^n where $T(\vec{x}) = A\vec{x}$

$\Rightarrow$ Then A is not invertible by IMT

$\therefore$ T is not onto $\mathbb{R}^n$ by IMT ∎