Name: Key

Math 362 Linear Algebra – Crawford

Quiz 1

12 September 2018

Books, calculators, and notes (in any form) are not are allowed. Show all your work for credit. ***Good luck!***

[Note: Each quiz score will be scaled to 15 points after grading.]

1. (2 pts) Compute the product below, if defined. If it is undefined, explain why.

$$\begin{bmatrix} -1 & 3 \\ 0 & -2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -1(4)+3(2) \\ 0(4)-2(2) \\ 2(4)+5(2) \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ -4 \\ 18 \end{bmatrix}}$$

3x2 ✓ 2x1

⇒ 3x1 ✓

2. (4 pts) Determine if $\mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$ is a linear combination of the columns of $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$.

$$\left[\begin{array}{ccc|c} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{cccc} 1 & -2 & 1 & 3 \\ -1 & 8 & 5 & 3 \\ 2 & 0 & 6 & 10 \end{array}\right]$$

$$\xrightarrow[-2R_1+R_3 \to R_3]{R_1+R_2 \to R_2} \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 6 & 6 & 6 \\ 0 & 4 & 4 & 4 \end{bmatrix} \xrightarrow[-\frac{2}{3}R_2+R_3 \to R_3]{\frac{1}{6}R_2 \to R_2} \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Note: Echelon Form is sufficient to answer question. There is not a row [0 0 0 ⋮ ■] So the system is consistent.

∴ $\vec{b}$ can be written as a linear combination of the columns of A.

But you may continue to RREF (same justification of answer)

$$\xrightarrow{2R_2+R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

3. (5 pts) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$.

For what value(s) of h is $\mathbf{y}$ in the plane spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$?

$$\begin{bmatrix} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{bmatrix} \xrightarrow[2R_1+R_3\to R_3]{-4R_1+R_2\to R_2} \begin{bmatrix} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & 8+h \end{bmatrix}$$

$$\xrightarrow{\frac{1}{5}R_2\to R_2} \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 3 & 8+h \end{bmatrix} \xrightarrow{-3R_2+R_3\to R_3} \left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 17+h \end{array}\right]$$

$\boxed{\text{If } h=-17, \text{ then } \vec{y} \text{ is in Span}\{\vec{v}_1, \vec{v}_2\}}$

4. (2 pts) Let A be a 3×2 matrix. Explain why the equation $A\mathbf{x} = \mathbf{b}$ cannot be consistent for all $\mathbf{b}$ in $\mathbb{R}^3$.

Since A has only 2 columns, it can have at most 2 pivot positions. Hence, there will be at least one row without a pivot. Hence a row of the form $[0 \ 0 \mid b]$ $b\neq 0$ is possible for some vectors $\vec{b}$ in $\mathbb{R}^3$ $\therefore$ $A\vec{x}=\vec{b}$ is not consistent for some $\vec{b}$ in $\mathbb{R}^3$.

5. (3 pts) Determine whether the following statements are TRUE or FALSE.

(a). One possible linear combination of the vectors $\mathbf{u}$ and $\mathbf{v}$ is the vector $-\frac{3}{2}\mathbf{u}$.

$\boxed{\text{TRUE}}$ $\quad -\frac{3}{2}\vec{u} + 0\vec{v}$

(b). If A is an $m\times n$ matrix and A has a pivot in every row, then the columns of A span $\mathbb{R}^m$.

$\boxed{\text{TRUE}}$ $\quad$ Theorem 4

(c). Span $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ contains only 3 vectors.

$\boxed{\text{FALSE}}$ $\quad$ It contains infinitely many vectors.