Name: ______________________________ **Exam 1**

Math 362 Linear Algebra – Crawford **05 October 2018**

Books and notes (in any form) are not are allowed. You may use a calculator – but please indicate when you use the matrix functions on the calculator. Put all of your work and answers on the provided paper and staple this cover sheet on top. Show all your work for credit. ***Good luck!***

Calculator # __________

1. (12 pts) Given the matrices, $A = \begin{bmatrix} 1 & 0 & -2 \\ 4 & -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}$ compute each of the following or explain why it is undefined.

(a). $\det(AB)$

(b). $B^T - A$

2. (14 pts) Given the matrix $A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & -8 \end{bmatrix}$ [Justify your answers.]

(a). Find a relationship between b_1, b_2, and b_3 such that the vector $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is in the span of the columns of A.

Give a geometric description of the span.

(b). Do the columns of A span $\mathbb{R}^3$?

(c). Do the columns of A span $\mathbb{R}^2$?

3. (10 pts) Determine whether the following vectors are linearly independent or dependent.

$$\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ 7 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ -1 \end{bmatrix}$$

4. (10 pts) Consider an economy with three sectors: Electricity, Water, and Coal. Electricity sells 24% of its output to Water, 32% to Coal, and retains the rest. Water sells 18% of its output to Electricity, 48% to Coal, and retains the rest. Coal sells 25% of its output to Electricity and 10% to Water and retains the rest.

(a). Construct an exchange table for this economy.

(b). Set up the system of equations that leads to equilibrium pricing for this economy. Write the system in matrix form, but ***but do not solve***.

5. (12 pts) Given $A = \begin{bmatrix} 2 & x \\ 3 & -2 \end{bmatrix}$,

(a). Find all value(s) of x, if any, for which A is not invertible.

(b). Find all value(s) of x, if any, so that $A = A^{-1}$.

6. (10 pts) Compute the following determinant by cofactor expansions. At each step, choose a row or column that involves the least amount of computation. [You must show the expansions and your work - no credit for just using the "det" button on your calculator.]

$$\begin{vmatrix} 3 & 2 & 0 & 1 \\ -1 & 2 & 5 & 0 \\ 2 & -2 & 0 & 3 \\ 0 & 1 & 0 & -4 \end{vmatrix}$$

7. (20 pts) Prove ***two*** of the following.
BONUS: You may do (or attempt) all three options and each will be graded out of 10 points. Whichever two you score the highest on will be your base grade. Any points from the other problem will be cut in third and added to your base grade. (e.g. If you get 10 points, 5 points, and 3 points, then your score will be $10 + 5 + \frac{3}{3} = 16$)

(a). (NEW) Suppose $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a linearly independent set in $\mathbb{R}^n$. Prove that $\{\mathbf{v}_1, \mathbf{v}_1 + \mathbf{v}_2\}$ is also linearly independent.

(b). (NOT NEW) Suppose that A is an invertible matrix. Prove that A^T is also invertible and that $(A^T)^{-1} = (A^{-1})^T$.

(c). (NOT NEW-ISH)Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Also suppose that B has no column of all zeros. If the second column in the product AB is a column of all zeros, prove that $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

8. (14 pts) Determine whether the following statements are true or false. If the statement is false, correct the statement and/or clearly explain why it is false.

(a). For all $\mathbf{x}$ in $\mathbb{R}^3$, $\mathbf{x}I_3 = \mathbf{x}$.

(b). For $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix}$, the determinant is given by $|A| = a\begin{vmatrix} f & g & h \\ j & k & l \end{vmatrix} - e\begin{vmatrix} b & c & d \\ j & k & l \end{vmatrix} + i\begin{vmatrix} b & c & d \\ f & g & h \end{vmatrix}$.

(c). If $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then $A\mathbf{x} = \mathbf{b}$ will have a unique for any $\mathbf{b}$. [Assume the sizes of $A, \mathbf{x}$ and $\mathbf{b}$ are such that the equations are defined.]

(d). If A and B are row equivalent $m \times n$ matrices and if the columns of A span $\mathbb{R}^n$, then so do the column of B.

(e). One possible linear combination of the vectors $\mathbf{u}$ and $\mathbf{v}$ is $\mathbf{0}$.